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Double-beta decay rates and nuclear matrix elements

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Obhajoba dizertačnej práce sa koná .... o .... h
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študijný odbor**: 4.1.2. Všeobecná fyzika a matematická fyzika**

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Contents

1 Introduction

Progress in neutrino physics has been impressive over recent decades, with the discovery of neutrino oscillations, which proves that neutrinos have mass. The best tool for addressing the origin of neutrino mass is the $0\nu\beta\beta$-decay of medium and heavy nuclei. This process, if it exists, would prove Majorana nature of neutrinos, and would measure the absolute scale of the neutrino mass spectrum. In addition, the $0\nu\beta\beta$-decay could give us a window on the non-standard model physics that gives rise to the tiny neutrino masses that we find in nature. It is worth mentioning that left-right symmetric theories are considered to be very appealing candidates for physics beyond the SM.

In this thesis the origin of the suppression of the double beta decay nuclear matrix elements (NMEs) is studied within an exactly solvable models expressed with generators of the SO(5) and SO(8) group. By using the advantage of the perturbation theory it is found that the two-neutrino double beta decay matrix elements do not depend on the mean field part of Hamiltonian and that they are governed by a weak violation of both SU(2) and SU(4) symmetries by the particle-particle interaction of Hamiltonian. The energy-weighted Fermi and Gamow-Teller sum rules connecting $\Delta Z = 2$ nuclei are proposed. It is suggested that these sum rules can be used to study the residual interactions of the nuclear Hamiltonian, which are relevant for charge changing nuclear transitions. Of primary interest is the process mediated by the exchange of light Majorana neutrinos interacting through the left-handed V-A weak currents. To our knowledge, for the first time, the corresponding formalism is extended by considering $p_{1/2}$-states of emitted electrons and recoil corrections to nucleon currents. The derived decay rate of the process is a sum of products of kinematical phase space factors and various nuclear matrix elements. By using exact Dirac wave function with finite nuclear size and electron screening numerical computation of phase space integrals is performed and the effect of the $p_{1/2}$ emission of electrons is evaluated. Next, the formalism of the $0\nu\beta\beta$-decay with the inclusion of the right-handed leptonic and hadronic currents and by assuming small neutrino masses is revisited by considering exact Dirac wave function of electron and higher order terms of nucleon current. The improved values of phase space integrals are presented and differential characteristics corresponding to various combinations of lepton number violating parameters are studied. Analysis in respect of the effective lepton number violating parameters due to right-handed currents are updated in light of recent progress achieved by the GERDA, EXO and KamlandZen experiments and by considering two types of published NMEs.

The main theoretical results obtained within this PhD thesis are summarized in following sections.

2 Fermi $2\nu\beta\beta$-decay matrix element within SO(5) model

We discuss the suppression mechanism of the double Fermi matrix element close to the point of restoration of isospin symmetry of the nuclear Hamiltonian in the context of residual nucleon-nucleon interaction. For the sake of simplicity we consider a schematic Hamiltonian, describing the gross properties of the β-decay processes in the simplest case of monopole Fermi transitions within the SO(5) model [1, 2]. In order to find explicit dependence of $M^{2\nu}_F$ on different parts of the nuclear Hamiltonian the perturbation theory is exploited.

2.1 Schematic Hamiltonian within the SO(5) model

In the model, protons and neutrons occupy only a single j shell. The Hamiltonian includes a single-particle term, proton-proton and neutron-neutron pairing, and a charge-dependent two-body interaction with both particle-hole and particle-particle channels as follows:

$$H = e_p N_p + e_n N_n - G_p S^{\dagger}_p S_p - G_n S^{\dagger}_n S_n + 2\chi\beta^-\beta^+ - 2\kappa P^- P^+, \tag{1}$$

where

$$N_i = \sum_m a^{\dagger}_{m,t_i} a_{m,t_i}, \quad \beta^- = \sum_m a^{\dagger}_{m,-\frac{1}{2}} a_{m,\frac{1}{2}},$$
$$S^{\dagger}_i = \frac{1}{2}\sum_m a^{\dagger}_{m,t_i}\tilde{a}^{\dagger}_{m,t_i}, \quad P^- = \sum_m a^{\dagger}_{m,-\frac{1}{2}}\tilde{a}^{\dagger}_{m,\frac{1}{2}}, \tag{2}$$

with i=p, n and $t_{n,p} = \pm 1/2$. $a^{\dagger}_{mt}$ (a_{mt}) is the creation (annihilation) operator of the single particle state $|jm, t>$ for protons and neutrons ($t = t_p, t_n$) and $\tilde{a}^{\dagger}_{mt} = (-1)^{j-m} a^{\dagger}_{-mt}$.

As we detailed discussed in thesis, in the Hamiltonian (1) isospin is not conserved in general. This is due to differences between proton and neutron pairing strengths and an arbitrary

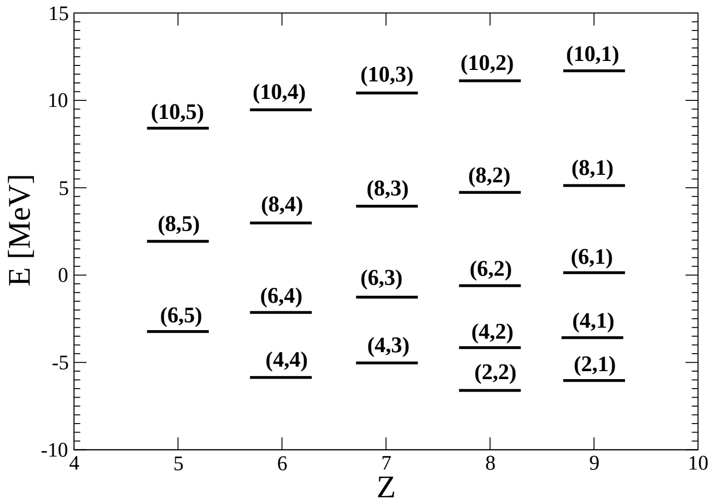

Figure 1: Energy of the 0^+ states of different isotopes are shown for $j = 19/2$ (and the set of parameters (4) with $4\kappa/G = 1$) in MeV vs. Z. States are labeled by (T, T_z).

strength of the proton-neutron isovector pairing component. However, particle number and isospin projection remain as good quantum numbers. The kth eigenstates of the Hamiltonian (1) with quantum numbers N and T_z can be expressed in terms of a basis labeled by a chain of irreducible representations of the SO(5) group, namely,

$$|k; NT_z\rangle = \sum_T c^{(k)}_{NTT_z} |NTT_z\rangle. \tag{3}$$

2.2 Double Fermi matrix element within perturbation theory

We shall assume a small violation of the isospin symmetry in particle-particle interaction part of Hamiltonian (1). For the numerical example we consider a large value of j to simulate the realistic situation corresponding to medium- and heavy-mass nuclei. The chosen parameters are given by

$$\begin{array}{lll} \Omega = 10, & N = 20, & 1 \leq T_z \leq 5, \\ e_p = 0.3 \text{ MeV}, & e_n = 0.1 \text{ MeV}, & G = 0.165 \text{ MeV}, \\ G_p = G_n = G, & \chi = 0.044 \text{ MeV}, & 0.7 \leq 4\kappa/G \leq 1.3. \end{array} \tag{4}$$

For $4\kappa/G = 1$ isospin symmetry is restored in particle-particle interaction part of Hamiltonian. In Fig. 1 we present 0^+ states with energy E_{TT_z} of different isotopes. This level scheme illustrates the situation for the $2\nu\beta\beta$-decay of ^{48}Ca. The isospin is known to be, to a very good approximation, a valid quantum number in nuclei. The ground states of ^{48}Ca and ^{48}Ti can be identified with $T = 4$ $T_z = 4$ and $T = 2$ $T_z = 2$, respectively; i.e., they are assigned into different isospin multiplets. As the total isospin projection lowering operator T^- is not changing the isospin the double Fermi matrix element $M^{2\nu}_F$ is non-zero only to the extent that the Coulomb interaction mixes the high-lying $T = 4$ T_z=2 analog of the ^{48}Ca ground state into the $T = 2$ $T_z = 2$ ground state of ^{48}Ti.

We shall study the double Fermi matrix element using perturbation theory within the discussed model close to a point of restoration of the isospin symmetry by the particle-particle interactions ($4\kappa/G = 1$). The isoscalar and isotensor terms of the Hamiltonian (1) represent the unperturbated and perturbated terms, respectively. We denote perturbated states and their energies with a superscript prime symbol ($|T'T_z\rangle$, E'_{TT_z}) unlike the states with a definite isospin ($|TT_z\rangle$, E_{TT_z}). For the transition $|4'4> \rightarrow |2'2>$ the double Fermi matrix element can be written as

$$M_F^{2\nu} = \sum_{T=4,6,8,10}^{10} \frac{\langle 2'2| T^- |T'3\rangle \langle T'3| T^- |4'4\rangle}{E'_{T3} - (E'_{44} - E'_{22})/2}. \quad (5)$$

It contains a sum over the states of the intermediate nucleus $|T'3\rangle$. However, up to second order of perturbation theory there is only a single contribution through the intermediate state $|4'3\rangle$. Thus, we have

$$M_F^{2\nu} \simeq \frac{\langle 2'2| T^- |4'3\rangle \langle 4'3| T^- |4'4\rangle}{E'_{33} - (E'_{44} - E'_{22})/2}. \quad (6)$$

The involved β-transition amplitudes are given by

$$\begin{aligned} \langle 4'3| T^- |4'4\rangle &= \langle 43| T^- |44\rangle \quad (7)\\ &\times \left(1 - \frac{1}{3}\frac{\Omega^2 (4\kappa - G)^2}{(44\chi + 22/3\,(G+2\kappa))^2}\left[\left|\langle 44| [A^\dagger \tilde{A}]^2_0 |64\rangle\right|^2 + \left|\langle 43| [A^\dagger \tilde{A}]^2_0 |63\rangle\right|^2\right]\right)\\ &+ \langle 63| T^- |64\rangle \frac{2}{3}\frac{\Omega^2 (4\kappa - G)^2}{(44\chi + 22/3\,(G+2\kappa))^2} \langle 64| [A^\dagger \tilde{A}]^2_0 |44\rangle \langle 63| [A^\dagger \tilde{A}]^2_0 |43\rangle \end{aligned}$$

and

$$\begin{aligned} \langle 2'2| T^- |4'3\rangle &= \langle 42| T^- |43\rangle \left[\sqrt{\frac{2}{3}}\Omega (G - 4\kappa) \frac{\langle 42| [A^\dagger \tilde{A}]^2_0 |22\rangle}{(28\chi + 14/3\,(G+2\kappa))}\right.\\ &+ \frac{2}{3}\frac{\Omega^2 (G - 4\kappa)^2}{(28\chi + 14/3\,(G+2\kappa))^2} \quad (8)\\ &\times \left.\left(\langle 42| [A^\dagger \tilde{A}]^2_0 |42\rangle \langle 42| [A^\dagger \tilde{A}]^2_0 |22\rangle - \langle 22| [A^\dagger \tilde{A}]^2_0 |22\rangle \langle 42| [A^\dagger \tilde{A}]^2_0 |22\rangle\right)\right]. \end{aligned}$$

The explicit values of matrix elements $\langle T'T'_z| [A^\dagger \tilde{A}]^t_m |TT_z\rangle$ are given in thesis.

If isospin symmetry is restored by particle-particle interaction part of Hamiltonian ($4\kappa = G$) we end up with $\langle 2'2| T^- |4'3\rangle = \langle 22| T^- |43\rangle = 0$. For the energy denominator in (6) we get

$$\begin{aligned} E'_{43} - \left(E'_{44} + E'_{22}\right)/2 &= 16\chi + \frac{7}{3}(G + 2\kappa) \quad (9)\\ &+\sqrt{\frac{1}{6}}\Omega (4\kappa - G) \left[2\langle 43| [A^\dagger \tilde{A}]^2_0 |43\rangle - \langle 44| [A^\dagger \tilde{A}]^2_0 |44\rangle - \langle 22| [A^\dagger \tilde{A}]^2_0 |22\rangle\right]\\ &+\frac{1}{3}\Omega^2 (4\kappa - G)^2 \left[\frac{\langle 64| [A^\dagger \tilde{A}]^2_0 |44\rangle^2}{44\chi + 22/3\,(G+2\kappa)} + \frac{\langle 42| [A^\dagger \tilde{A}]^2_0 |22\rangle^2}{28\chi + 14/3\,(G+2\kappa)} - 2\frac{\langle 63| [A^\dagger \tilde{A}]^2_0 |43\rangle^2}{44\chi + 22/3\,(G+2\kappa)}\right]. \end{aligned}$$

We note that the energy denominator $E'_{43} - (E'_{44} - E'_{22})/2$ nor the whole double Fermi matrix element $M_F^{2\nu}$ depend explicitly on the mean-field parameters e_p and e_n.

In Fig. (2) $M_F^{2\nu}$ is plotted as function of ratio $4\kappa/G$. We see that results obtained with second- order perturbation theory agree well with exact results within a large range of this parameter.

2.3 Conclusions

The suppression mechanism of the double Fermi matrix element close to the point of restoration of isospin symmetry of the nuclear Hamiltonian was studied in exactly solvable model expressed with generators of the SO(5) group in which protons and neutrons occupy only a single j shell. This model reproduces the main features of the results obtained in realistic calculations and remains a useful tool for understanding different nuclear physics phenomena. The main results are follows:

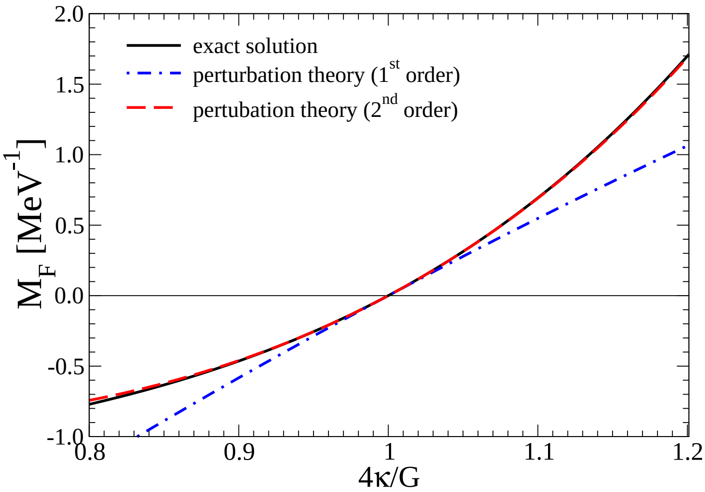

Figure 2: Matrix element $M_F^{2\nu}$ for the double-Fermi two-neutrino double-beta decay mode as a function of the ratio $4\kappa/G$ for a set of parameters (4). Exact results are indicated with a solid line. The results obtained within the perturbation theory up to the first and second order in isotensor contribution to Hamiltonian are shown with dash-dotted and dashed lines, respectively. The restoration of isospin symmetry by particle-particle interaction part of Hamiltonian is achieved for $4\kappa/G = 1$.

- By using perturbation theory an explicit dependence of the two-neutrino double-beta decay matrix element on the components of nuclear Hamiltonian was found by assuming a weak violation of isospin symmetry due to difference between strengths of like nucleon pairing and proton-neutron isovector pairing.
- It was found that the mean-field part of the Hamiltonian does not enter explicitly to the double-Fermi matrix element and is related only to the calculation of unperturbated states of the Hamiltonian.
- There is a dominance of double-Fermi transition through a single state of intermediate nucleus.

3 Fermi and Gamow-Teller $2\nu\beta\beta$-decay matrix element within SO(8) model

Being encouraged by results obtained for double-Fermi transitions a simultaneous study of the two-neutrino double-beta Gamow-Teller and Fermi matrix elements is performed within an exactly solvable model, which allows a violation of both spin-isospin SU(4) and isospin SU(2) symmetries, and is expressed with generators of the SO(8) group. The motivation is the fact that the isospin is known to be a good approximation in nuclei unlike the Wigner spin-isospin SU(4) symmetry, which is strongly broken by the spin-orbit splitting in heavy nuclei. A subject of interest is also a connection to the realistic calculations performed within the quasiparticle random phase approximation with a restoration of the isospin symmetry.

3.1 Schematic Hamiltonian expressed with generators of SO(8) group

The Hamiltonian of the model is an extension of the Hamiltonian introduced in [3], and posses the main qualitative features of a realistic Hamiltonian relevant to double beta decay. It contains proton and neutron single-particle terms and the two-body residual interaction, which components are isovector spin-0, isoscalar spin-1 pairing and the particle-hole force in the T=1 S=1 channel. We have

$$\begin{aligned} H &= e_n N_n + e_p N_p + g_{ph} \sum_{a,b} E^\dagger_{a,b} E_{a,b} \\ &- g_{pair} \left(\sum_{M_T=-1,0,1} A^\dagger_{0,1}(0,M_T) A_{0,1}(0,M_T) + \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0) A_{1,0}(M_S,0) \right) \\ &+ \underbrace{(g_{pair} - g^{T=0}_{pp}) \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0) A_{1,0}(M_S,0) + (g_{pair} - g^{T=1}_{pp}) A^\dagger_{0,1}(0,0) A_{0,1}(0,0)}_{H_I} . \end{aligned} \tag{10}$$

Here, g_{pair}, $g^{T=1}_{pp}$,$g^{T=0}_{pp}$ and g_{ph} denote the strengths of the isovector like nucleon spin-0 pairing ($L = 0, S = 0, T = 1, M_T \pm 1$), isovector proton-neutron spin-0 pairing ($L = 0, S = 0, T = 1, M_T = 0$), isoscalar spin-1 pairing ($L = 0, S = 1, T = 0$) and particle-hole force ($L = 0, S = 1, T = 1$), respectively. The proton number operator N_p, neutron number operator N_n, particle-particle operators $A^\dagger_{S,T}(M_S, M_T)$ and particle-hole GT operators $E_{a,b}$ are defined as [4]

$$\begin{aligned} A^\dagger_{S,T}(M_S, M_T) &= \sum_{l,m,m_s,m_t} \sqrt{l+\frac{1}{2}} C^{00}_{lmlm'} C^{TM_T}_{\frac{1}{2}m_t \frac{1}{2}m'_t} C^{SM_S}_{\frac{1}{2}m_s \frac{1}{2}m'_s} a^\dagger_{lmm_sm_t} a^\dagger_{lm'm'_sm'_t}, \\ (S,T) &= (0,1) \text{ or } (1,0), \end{aligned} \tag{11}$$

the particle-hole GT operators take the form

$$E_{a,b} = \sum_{l,m,m_s,m_t} \langle (m_s+a)(m_t+b)|\, \sigma_a \tau_b \,|m_s m_t\rangle \, a^\dagger_{lm(m_s+a)(m_t+b)} a_{lmm_sm_t}, \tag{12}$$

and particle number operators are written as

$$N_i = \sum_{l,m,m_s,m,l} a^\dagger_{lmm_sm_{ti}} a_{lmm_sm_{ti}}, \quad i = p, n, \quad m_{t_n,t_p} = \pm 1/2. \tag{13}$$

Here, we consider a set of single-particle states with the associated creation and annihilation operators, $a^\dagger_{lmm_sm_t}$ and $a_{lmm_sm_t}$, which are labeled by orbital angular momentum l its projection on z axis m, and z components of spin ($m_s = 1/2$) and isospin ($m_t = 1/2$). The one-particle operators σ_a and τ_b represent spherical components of the single-particle Pauli spin and isospin operators with convention used in [4].

The six particle-particle operators $A^\dagger_{S,T}(M_S, M_T)$ and their hermitian conjugates together with nine particle-hole GT operators $E_{a,b}$, total spin $\vec{S}$ and isospin $\vec{T}$ operators and total particle number operator (defined for convention as $Q_0 = \Omega - \frac{1}{2}(N_p + N_n)$) represent 28 operators which generate the group SO(8) [4]. We note that matrix elements of generators SO(8) group are known in this basis [4, 5].

The Hamiltonian (10) is decomposed in two parts: H_0 (first two lines of (10)), the unperturbed Hamiltonian, and H_I, the perturbing one. The eigenstates of unpertubated Hamiltonian H_0 are characterized by the number of nucleon pairs $N/2$ (only systems with even number of nucleons are considered), the isospin T, T_z spin S, S_z and a quantum number n corresponding to the irreducible SU(4) representation $[n, n, 0]$. The single-particle and particle-hole interaction components of H_0 violate both isospin and spin-isospin symmetries and as a consequence energies of states with the same quantum numbers T and n are different for a given $T_z = (N_n - N_p)/2$ ($T_z \equiv M_T$). If $g_{pair} = g^{T=0}_{pp}$ and $g_{pair} = g^{T=1}_{pp}$ the isospin

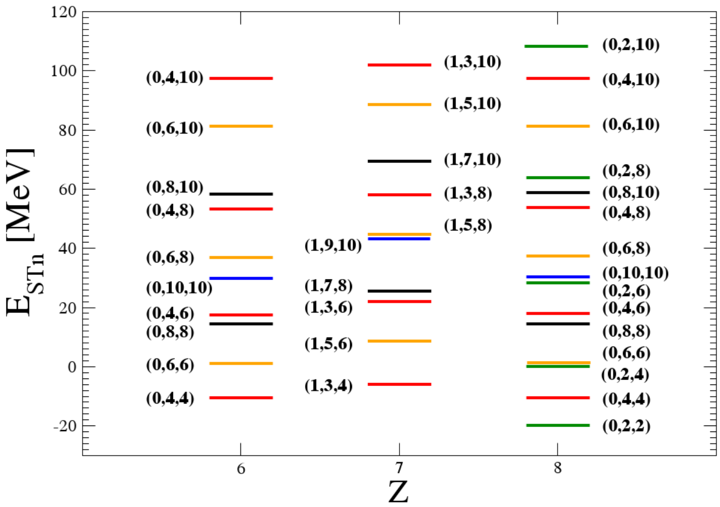

Figure 3: Eigenenergies E_{STn} of the Hamiltonian (10) for set of parameters (15), Z = 6, 7 and 8 ($Z = N/2 - M_T$, M_T=4, 3 and 2) and by assuming $H_I = 0$. Energy states are labeled by spin, isospin and the SU(4) quantum number n: (S,T,n). The levels with different value of $S + T$ are displayed in different color: S+T= 2 (green), 4 (red), 6 (orange), 8 (black) and 10 (blue).

and spin-isospin symmetries of particle-particle interaction are restored we get $H = H_0$ and $H_I = 0$. If $g_{pair} \neq g_{pp}^{T=0}$ and/or $g_{pair} \neq g_{pp}^{T=1}$, the Hamiltonian (10) is not more diagonal with respect to quantum numbers T and n. The eigenstate of the Hamiltonian $|S, M_S, T', M_T, n'\rangle$ can be expressed with eigenstates of unperturbated Hamiltonian H_0 as follows:

$$\left|S, M_S, T', M_T, n'\right\rangle = \sum_{n,T} c^{T'n'}_{S,M_S,T,M_T,n} \left|S, M_S, T, M_T, n\right\rangle . \tag{14}$$

For the numerical application we consider a set of parameters as follows [3]:

$$\begin{aligned} &e_p = 1.2MeV \qquad e_n = 1.1MeV \qquad \Omega = 12, \\ &N = 20, \qquad g_{pair} = 0.5MeV, \qquad g_{ph} = 1.5\ g_{pair}. \end{aligned} \tag{15}$$

In Fig. 3 we present states with energy E_{STn} of different isotopes. The considered level scheme illustrates the situation with double GT transition for ^{48}Ca. Within the studied model in the SU(4) symmetry limit the ground states of ^{48}Ca and ^{48}Ti can be identified with S=0 T=4 T_z=4 n=4 and S=0 T=2 T_z=2 n=2, respectively, and the intermediate states in ^{48}Sc with S=1 (T=3,5,7, and 9) T_z=3 (n=4, 6, 8 and 10). As the GT operator is not changing quantum number n the double GT matrix elements connecting initial and final ground states is nonzero only to the extent the breaking of SU(4) symmetry mixes the high-lying (0,4,4) analog of the ^{48}Ca ground state into (0,2,2) analog of the ^{48}Ti.

3.2 The GT matrix element in the case of isospin symmetry

We consider a small violation of the SU(4) spin-isospin symmetry in nuclear Hamiltonian (10) due to $g_{pair} \neq g_{pp}^{T=0}$ and that isospin is a good quantum number, i.e, $g_{pair} = g_{pp}^{T=1}$, what implies $M_F^{2\nu} = 0$.

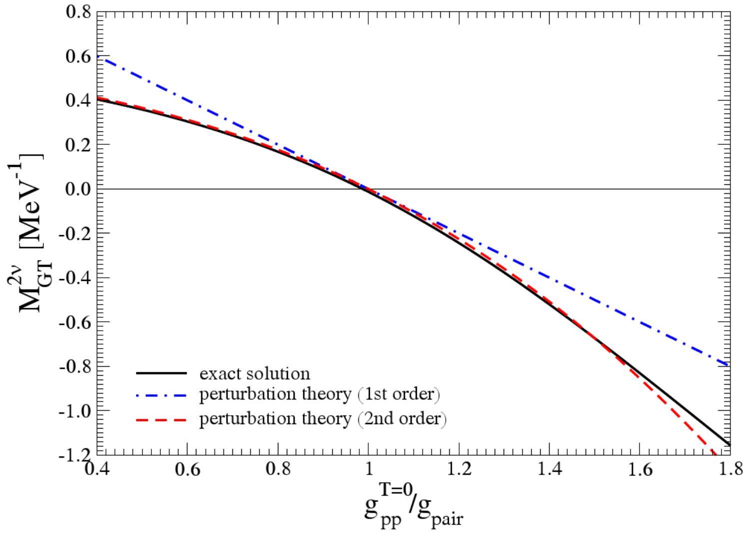

Figure 4: Matrix element $M_{GT}^{2\nu}$ for the double-GT two-neutrino double-beta decay mode as function of ratio $g_{pp}^{T=0}/g_{pair}$ for a set of parameters (4). Exact results are indicated with a solid line. The results obtained within the perturbation theory up to the first and second order in H_I contribution to Hamiltonian (10) are shown with dashed-dotted and dashed lines, respectively. The restoration of spin-isospin symmetry of particle-particle interaction is achieved for $g_{pp}^{T=0}/g_{pair} = 1$.

As an example we discuss in details the GT matrix element for $2\nu\beta\beta$ decay from the state with $S = 0$, $T = M_T = 4$ to the state with $S = 0$, $T = M_T = 2$. The corresponding transition is

$$\left|0, 0, 4, 4, 4'\right\rangle \to \left|1, M_S, 3, 3, 4'\right\rangle \to \left|0, 0, 2, 2, 2'\right\rangle . \quad (16)$$

By keeping in mind a small violation of SU(4) symmetry we denote perturbated states and their energies with a superscript prime symbol ($|S, M_S, T, M_T, n'\rangle$, $E'_{S,M_S,T,M_T,n}$) unlike the states with a definite quantum number n ($|S, M_S, T, M_T, n\rangle$, $E_{S,M_S,T,M_T,n}$).

Up to the second order of parameter $(g_{pair} - g_{pp}^{T=0})$ we obtain the double GT matrix element as

$$M_{GT}^{2\nu} = \sum_{n'} \frac{\langle 022'|\, \vec{\sigma}\tau^- \,|13n'\rangle \cdot \langle 13n'|\, \vec{\sigma}\tau^- \,|044'\rangle}{E'_{13n} - \left(E'_{044} - E'_{022}\right)/2} \simeq \frac{\langle 022'|\, \vec{\sigma}\tau^- \,|134'\rangle \cdot \langle 134'|\, \vec{\sigma}\tau^- \,|044'\rangle}{E'_{134} - \left(E'_{044} - E'_{022}\right)/2}. \quad (17)$$

The allowed intermediate states $|13n'\rangle$ are those with S=1, T=3 and $n' = 4, 6, 8$ and 10. We note that up to second order of perturbation theory there is only a single contribution through the intermediate state $|134'\rangle$ and the product of two corresponding β-amplitudes (numerator of (17)) takes the form

$$\left\langle 022' \left|\vec{\sigma}\tau^-\right| 134'\right\rangle \cdot \left\langle 134' \left|\vec{\sigma}\tau^-\right| 044'\right\rangle = 144\sqrt{\frac{231}{35}} \left(\frac{(g_{pair} - g_{pp}^{T=0})}{10 g_{pair} + 20 g_{ph}} - \frac{267 (g_{pair} - g_{pp}^{T=0})^2}{35 (10 g_{pair} + 20 g_{ph})^2} \right).$$

and for the energy denominator in (17) we obtain

$$E'_{134} - \frac{E'_{022} + E'_{044}}{2} = 5 g_{pair} + 9 g_{ph} + (g_{pair} - g_{pp}^{T=0}) \frac{39}{5} + \frac{(g_{pair} - g_{pp}^{T=0})^2}{g_{pair} + 2 g_{ph}} \left(\frac{1249263}{171500} \right).$$

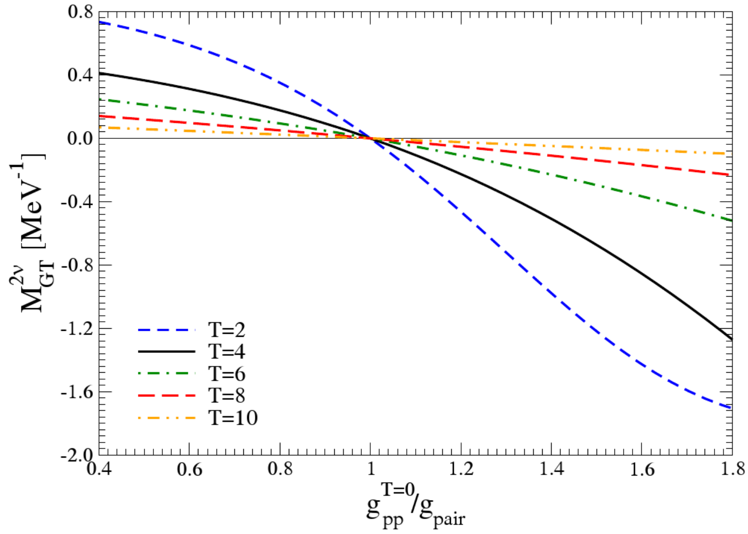

Figure 5: Matrix element $M^{2\nu}_{GT}$ for the double-GT two-neutrino double-beta decay mode as function of ratio $g^{T=0}_{pp}/g_{pair}$ for different initial state with $T = M_T$ (M_T=2, 4, 6, 8 and 10).

It is worth to notice that neither the numerator nor denominator of $M^{2\nu}_{GT}$ depend explicitly on the single-particle energies e_n and e_p.

In Fig.4 $M^{2\nu}_{GT}$ is plotted as function of ratio $g^{T=0}_{pp}/g_{pair}$. We see that results obtained within the second order perturbation theory agree well with exact results within a large range of this parameter. For $g^{T=0}_{pp}/g_{pair} = 1$ the restoration of the SU(4) symmetry of particle-particle interaction is achieved, i.e., $M^{2\nu}_{GT}$ is equal to zero.

Usually, ground states of stable even-even nuclei are identified with isospin $T = T_z$. The dependence of $M^{2\nu}_{GT}$ on the isospin of the initial nucleus is presented in Fig.5. We see that for a fixed value of $g^{T=0}_{pp}/g_{pair}$ (i.e., breaking of the SU(4) symmetry) the absolute value of $M^{2\nu}_{GT}$ decreases with increasing isospin T. We note that apart from the shell effects of magic nuclei this tendency is observed also for measured $2\nu\beta\beta$-decay matrix elements [6].

3.3 The Fermi and GT matrix elements in the case of broken SU(2) and SU(4) symmetries

The main task to be addressed in this subsection is what is the dependence of $M^{2\nu}_{F}$ and $M^{2\nu}_{GT}$ on both quantities $g^{T=1}_{pp}/g_{pair}$ and $g^{T=0}_{pp}/g_{pair}$. Recall that $g^{T=1}_{pp} \neq g_{pair}$ breaks both the SU(2) isospin and the SU(4) spin-isospin symmetries of particle-particle interaction unlike $g^{T=0}_{pp} \neq g_{pair}$, which is associated only with the violation of the SU(4) symmetry.

We shall consider the $2\nu\beta\beta$-decay transition from the initial $|04'4'\rangle$ to final $|02'2'\rangle$ ground state. Up to the first order in the perturbation theory for double Fermi and GT matrix elements we find

$$M^{2\nu}_{F} = -\frac{48\sqrt{\frac{33}{5}}\left(g_{pair} - g^{T=1}_{pp}\right)}{(5g_{pair} + 3g_{ph})(10g_{pair} + 6g_{ph})}, \quad (18)$$

$$M^{2\nu}_{GT} = \frac{144\sqrt{\frac{33}{5}}}{5g_{pair} + 9g_{ph}}\left\{\frac{(g_{pair} - g^{T=0}_{pp})}{(10g_{pair} + 20g_{ph})} + \frac{2g_{ph}(g_{pair} - g^{T=1}_{pp})}{(10g_{pair} + 20g_{ph})(10g_{pair} + 6g_{ph})}\right\}. \quad (19)$$

We see that $M^{2\nu}_{F}$ depends only on strength of the isovector interaction $g^{T=1}_{pp}$ unlike $M^{2\nu}_{GT}$,

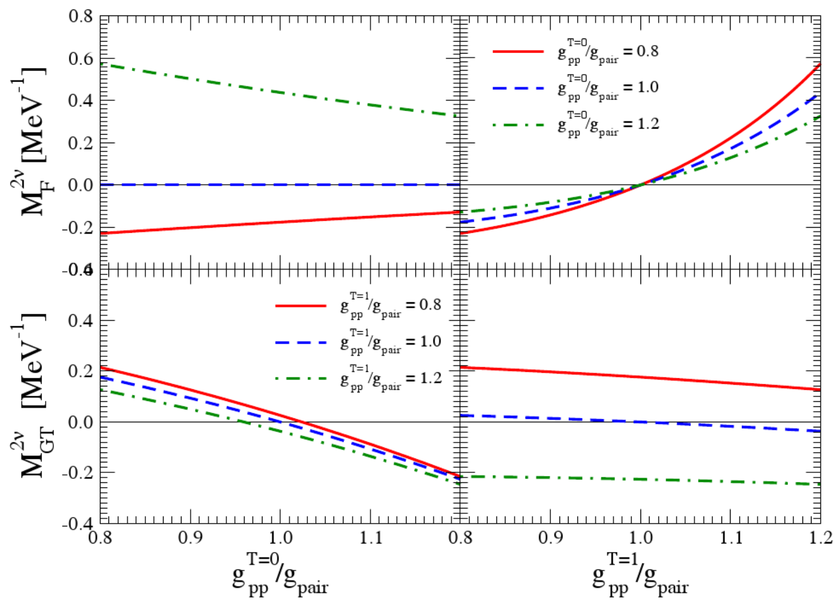

Figure 6: Matrix elements $M_F^{2\nu}$ and $M_{GT}^{2\nu}$ as function of of ratios $g_{pp}^{T=0}/g_{pair}$ and $g_{pp}^{T=1}/g_{pair}$ for transition from the initial $|04'4'\rangle$ to final $|02'2'\rangle$ ground state and a set of parameters (4). The results are obtained within the perturbation theory up to the second order.

which depends also on the strength of the isoscalar interaction $g_{pp}^{T=0}$. Due to the violation of the isospin symmetry the final ground state $|02'2'\rangle$ is mixed with both first $|04'4'\rangle$ and second $|02'4'\rangle$ excited states (see Fig. 1) resulting in $g_{pp}^{T=1}$ contribution to $M_{GT}^{2\nu}$.

In Fig. 6 we present behavior of $M_F^{2\nu}$ and $M_{GT}^{2\nu}$ as function of $g_{pp}^{T=1}/g_{pair}$ ($g_{pp}^{T=0}/g_{pair}$) for a particular values of $g_{pp}^{T=0}/g_{pair}$ ($g_{pp}^{T=1}/g_{pair}$). Results were obtained within the perturbation theory up to the second order. We see clearly that for $g_{pp}^{T=1}/g_{pair} = 0$ matrix element $M_F^{2\nu}$ does not depend on $g_{pp}^{T=0}$ and varies strongly with change of $g_{pp}^{T=1}$. A different behavior offer $M_{GT}^{2\nu}$, which weakly depends on the $g_{pp}^{T=1}$ and significantly on the $g_{pp}^{T=0}$. These conclusions agree qualitatively well with results obtained for two-neutrino double-beta decay transitions within the proton-neutron QRPA with restoration of the isospin symmetry [7]. The advantage of the study within considered schematic model and in perturbation theory is that explicit dependence of $M_F^{2\nu}$ and $M_{GT}^{2\nu}$ on isoscalar and isovector strength of particle-particle interactions can be determined.

3.4 Conclusions

The anatomy of the two-neutrino double beta decay matrix elements was studied within a schematic model, which can be solved exactly and yet contains most of the qualitative features of a more realistic description, and by taking advantage of the perturbation theory. The main results are follows:

- By using perturbation theory an explicit dependence of the two-neutrino double-beta decay matrix elements on the like-nucleon pairing, particle-particle T=0 and T=1, and particle-hole proton-neutron interactions was obtained. It was found that double beta decay matrix elements do not depend on the mean field part of Hamiltonian and that they are governed by a weak violation of both SU(2) and SU(4) symmetries by the particle-particle interaction of Hamiltonian. It is shown that the quantity $2E_n -$

$E_i - E_f$ depends only on the particle-particle and particle-hole components of residual interaction of Hamiltonian addressing the point that two single beta decays in nucleus must be correlated by means of nucleon-nucleon interaction.

- It was found that the mean-field part of the Hamiltonian does not enter explicitly in the decomposition of the double Gamow-Teller matrix element and is related only to the calculation of unperturbated states of the Hamiltonian. This fact might be an explanation for a smallness of this matrix element being governed by a small violation of the SU(4) symmetry by the particle-particle interaction of the Hamiltonian, in spite of the fact that the SU(4) symmetry is strongly broken by the mean field due to the spin-orbit splitting.
- The obtained results within schematic model justify realistic calculation within the QRPA with restoration of isospin symmetry, namely that the double Fermi matrix element depends strongly on the isovector part of the particle-particle neutron-proton interaction, unlike the double Gamow-Teller matrix element, which depends strongly on its isoscalar part.
- For fixed value of the residual interaction it was shown that the value of double Gamow-Teller matrix element decreases by an increase of the isospin of the initial ground state. This tendency is found also in the case of measured double Gamow-Teller matrix elements being partially spoiled by different pairing properties of semi-magic and magic nuclei.
- It was showed that double Gamow-Teller matrix element contains a small component due to a violation of the isospin symmetry. By keeping in mind that in nuclear physics the isospin symmetry is conserved to a great extent it is recommended for evaluation of double-beta decay matrix elements to use many-body approaches with a conservation or restoration of the isospin symmetry (the QRPA and IBM with restoration of isospin symmetry).

4 Light Majorana neutrino mass mechanism of the neutrinoless double-beta decay with emission of s and p electrons

We extend the formalism of the Majorana neutrino mass mechanism of the neutrinoless double-beta by considering $p_{1/2}$-states of emitted electrons and recoil corrections to nucleon currents. In thesis, we derived the improved formula for the half-life of the $0\nu\beta\beta$-decay as a sum of products of kinematical phase space factors and various nuclear matrix elements

$$\begin{aligned}\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} &= \frac{|m_{\beta\beta}|^2}{m_e^2} g_A^4 \left(2Re\left\{M_s M_r^*\right\} G_{sr} + 2Re\left\{M_s M_p^*\right\} G_{sp} + 2Re\left\{M_r M_p^*\right\} G_{rp}\right. \\ &+ \left. G_{ss}\left|M_s\right|^2 + G_{rr}\left|M_r\right|^2 + G_{pp}\left|M_p\right|^2\right), \end{aligned} \tag{20}$$

where G_{ij} (i,j=s,r,p) are phase-space integrals as follows:

$$G_{ij} = \frac{G_\beta^4 m_e^7}{32\pi^5 R^2 \ln 2} \frac{1}{m_e^5} \int_{m_e}^{E_i - E_f - m_e} g_{ij}\ \varepsilon_1 p_1\ \varepsilon_2 p_2\ d\varepsilon_1 \tag{21}$$

with $\varepsilon_2 = E_i - E_f - \varepsilon_1$, $p_i = \sqrt{\varepsilon_i^2 - m_e^2}$ (i=1,2), and $G_\beta = G_F \cos\theta_C$, where G_F and θ_C are Fermi constant and Cabbibo angle, respectively. The factors g_{ij} have a form

$$\begin{aligned}
g_{ss} &= C_{ss}(\varepsilon_1)C_{ss}(\varepsilon_2), \quad g_{sp} = -C_{sp}(\varepsilon_1)C_{sp}(\varepsilon_2), \\
g_{pp} &= C_{pp}(\varepsilon_1)C_{pp}(\varepsilon_2), \\
g_{rr} &= C_{ss}(\varepsilon_1)C_{pp}(\varepsilon_2) + C_{pp}(\varepsilon_1)C_{ss}(\varepsilon_2) - 2g_{sp}, \\
g_{sr} &= C_{ss}(\varepsilon_1)C_{sp}(\varepsilon_2) + C_{sp}(\varepsilon_1)C_{ss}(\varepsilon_2), \\
g_{rp} &= -C_{sp}(\varepsilon_1)C_{pp}(\varepsilon_2) - C_{pp}(\varepsilon_1)C_{sp}(\varepsilon_2.
\end{aligned} \tag{22}$$

Here, $C_{..}$ are combinations of radial components of $s_{1/2}$ and $p_{1/2}$ wave functions,

$$\begin{aligned}
C_{ss}(\varepsilon) &= g_{-1}^2(\varepsilon) + f_{+1}^2(\varepsilon), \quad C_{pp}(\varepsilon) = g_{+1}^2(\varepsilon) + f_{-1}^2(\varepsilon), \\
C_{sp}(\varepsilon) &= g_{-1}(\varepsilon)f_{-1}(\varepsilon) - g_{+1}(\varepsilon)f_{+1}(\varepsilon).
\end{aligned} \tag{23}$$

M_s and M_p are nuclear matrix elements associated with emission of $s_{1/2}$ and $p_{1/2}$ electrons, respectively. M_r is due to nucleon recoil and emission of $p_{1/2}$ electron. Explicit form of matrix elements is given in thesis. We note that commonly written standard formula for the half-life of the $0\nu\beta\beta$-decay is equivalent with our derived formula (20), when all terms except that proportional to the G_{ss} vanish.

Table 1: Phase-space factors G_{ij} $[10^{-18} yr^{-1}]$ (21)-(22) obtained using of exact Dirac wave functions of electrons with finite nuclear size and electron screening.

	$Q_{\beta\beta}[MeV]$	G_{ss}	G_{sr}	G_{rr}	G_{sp}	G_{rp}	G_{pp}
^{48}Ca	4.27226	24 834.	−4 138.3	690.26	−171.01	28.553	1.1824
^{76}Ge	2.03904	2 368.1	−529.26	118.37	−29.513	6.6047	0.36878
^{82}Se	2.99512	10 176.	−2 499.4	614.25	−152.98	37.619	2.3055
^{96}Zr	3.35037	20 621.	−5 929.3	1 705.7	−424.86	122.29	8.7718
^{100}Mo	3.03440	15 953.	−4 738.2	1 407.9	−350.88	104.31	7.7325
^{110}Pd	2.01785	4 828.5	−1 504.8	469.16	−117.07	36.518	2.8437
^{116}Cd	2.8135	16 734.	−5 569.5	1 854.5	−462.44	154.05	12.802
^{124}Sn	2.28697	9 063.5	−3 082.8	1 049.0	−261.74	89.101	7.5711
^{130}Te	2.52697	14 255.	−5 071.1	1 804.7	−450.22	160.29	14.242
^{136}Xe	2.45783	14 619.	−5 385.7	1 984.9	−495.23	182.59	16.803
^{150}Nd	3.37138	63 163.	−26 409.	11 045.	−2 754.1	1 152.3	120.25

4.1 Calculation and discussion

The phase space factors G_{ij} $(ij = ss, pp, rr, sp, sr, rp)$ (see Eq. (21)) are numerically calculated for nuclei of experimental interest. The input parameters are Q-value, nuclear radius and radial electron wave functions at nuclear radius, which are evaluated by means of the subroutine package RADIAL [8]. It is assumed the homogeneous electric charge distribution inside a nucleus and the screening of atomic electrons. The calculated phase-space factors are presented in the Table 1. We see that from the phase space factors, which are due to the $p_{1/2}$ wave of electrons, the largest is G_{sr}. For medium-heavy nuclei its absolute value is only by about factor 2-3 smaller when compared with the dominant phase-space factor G_{ss}. The third largest is the phase-space factor G_{rr}, which has also origin in the nucleon recoil. Other kinematical factors G_{sp}, G_{rp} and G_{pp} are significantly suppressed in comparison with G_{ss}, at least by about factor 80-20.

Table 2: Qualitative evaluation of Δ (see Eq. (24)), namely the effect of contributions associated with emission of at least one electron in the $p_{1/2}$ wave state to the decay rate. The values of matrix elements due to $p_{1/2}$-wave electrons and nucleon recoil are estimated by a comparison of involved two-nucleon exchange potentials for r = 1 and 2 fm, i.e., for a relative distance of decaying neutrons in the nucleus known to give maximal contribution to matrix elements. The important role in evaluation of the effect plays a suppression of potential associated with nucleon recoil $h_R(r)$ in respect to the potential $h_{GT}(r)$ entering the largest matrix element M_{GT}).

	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd	^{116}Cd	^{124}Sn	^{130}Te	^{136}Xe	^{150}Nd
					r = 1 fm						
$2\frac{h_R(r)}{h_{GT}(r)}$	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.03
$\Delta[\%]$	0.31	0.55	0.93	1.81	2.03	2.44	4.22	3.24	3.70	4.10	5.74
					r = 2 fm						
$2\frac{h_R(r)}{h_{GT}(r)}$	0.71	0.07	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05
$\Delta[\%]$	1.19	0.46	0.20	0.60	0.82	1.21	3.60	1.95	2.37	2.75	4.27

At present, the matrix elements M_s from reasonable calculations differ from one another by up to factors of 3 and the desired accuracy is of the order of 20-30 %. The corresponding uncertainty of the theoretical decay rate apart from the unknown value of $m_{\beta\beta}$ is by up to factor 9 and the desired accuracy is about 40-60 %. By keeping this fact in mind we just estimate the effect of emitted $p_{1/2}$ electrons on the $0\nu\beta\beta$-decay rate. The main difference among absolute values of different nuclear matrix elements is due to involved two-nucleon exchange potentials. We estimate the value of matrix elements M_r and M_p relative to M_s by assuming two approximations: i) We average potentials over all directions of $\mathbf{r}_+$ by keeping in mind translational invariance of any potential. ii) The value of additional nuclear matrix elements of Fermi, Gamow-Teller and tensor types is determined by a ratio of involved potentials to those potentials entering the dominant matrix elements M_F, M_{GT} and M_T for r = 1 and 2 fm, respectively. Recall that the dominant contribution to nuclear matrix elements is for about this internucleon distances [9]. Above mentioned approximations result in following relations $M_p = M_s$ and $M_r = -2\frac{h_R(r)}{h_{GT}(r)} M_s$.

The effect of $p_{1/2}$ electron contributions to $0\nu\beta\beta$-decay rate is determined by the quantity Δ, which is given by

$$\Delta = \left| \frac{\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} - \left[T_{1/2}^{0\nu\beta\beta}\right]_{ss}^{-1}}{\left[T_{1/2}^{0\nu\beta\beta}\right]_{ss}^{-1}} \right| \times 100\%. \quad (24)$$

Here, $\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1}$ and $\left[T_{1/2}^{0\nu\beta\beta}\right]_{ss}^{-1}$ stand for the half-lives given in Eqs. (20) and s-wave contribution only (term proportional to G_{ss} in (20)), respectively. In Table 2 the calculated values of Δ for nuclei of experimental interest are presented. We see that the largest effect of emitted electrons in $p_{1/2}$-wave states is for nuclei with largest number of protons and its value is below 6 %.

4.2 Conclusion

An improved expression for the $0\nu\beta\beta$-decay rate, which accounts for effects of the $p_{1/2}$-waves of emitted electrons and of the nucleon recoil, was presented. In the derivation the exact relativistic electron wave functions at nuclear radius were considered, what allowed to express the $0\nu\beta\beta$decay rate as a combination of 6 kinematical phase-space factors (G_{ss}, G_{sr}, G_{sp}, G_{pp}, G_{rp}, and G_{rr}) and three nuclear matrix elements (M_s, M_p, and M_r). The phase-space factors were calculated by using numerical solution of Dirac equation for radial electron wave function with finite nuclear size and electron screening. The absolute value of the phase-space factor G_{sr} is found to be suppressed by only factor of 3 in comparison to the dominant phase-space factor G_{ss} associated with the emission of only electrons in $s_{1/2}$ state. The values of unknown nuclear matrix elements were estimated by using a comparison of involved two-nucleon exchange potentials at internucleon distance of 1 and 2 fm. We conclude that the effect of emitted $p_{1/2}$ electrons on the $0\nu\beta\beta$-decay rate is below 10 %.

5 Phase-space factors for the neutrinoless double-beta decay with the right-handed currents

While the light-neutrino mass mechanism of the $0\nu\beta\beta$-decay described in section 4 and resulted in half-life (20) is certainly the most prominent realization of this decay, Majorana neutrino masses are not the only element of beyond Standard Model physics which can induce it. There exist a large range of new physics models that incorporate the lepton number violation and which lead to the potentially observable $0\nu\beta\beta$-decay rates [10]. One of them is the left-right symmetric model which induces new left-right and right-right current interactions in the standard β-decay Hamiltonian.

The main aim of this chapter is to rederive the half-life for the $0\nu\beta\beta$-decay with extension to the right-handed leptonic and hadronic currents and calculate the involving sphase-space factors more accurate by using exact Dirac wave functions with finite nuclear size and electron screening.

5.1 Decay rate for the neutrinoless double-beta decay with inclusion of right-handed currents

The most general form of the weak β-decay Hamiltonian inducing the $0\nu\beta\beta$-decay with inclusion of right-handed leptonic j_R and hadronic currents J_R has a form [11]

$$H^{\beta} = \frac{G_{\beta}}{\sqrt{2}} \left[j_L^{\rho} J_{L\rho}^{\dagger} + \chi\, j_L^{\prime\rho} J_{R\rho}^{\dagger} + \eta\, j_R^{\prime\rho} J_{L\rho}^{\dagger} + \lambda\, j_R^{\rho} J_{R\rho}^{\dagger} + h.c. \right] . \tag{25}$$

The derived half-life for the $0\nu\beta\beta$-decay with right hadronic and leptonic currents can be written as

$$\begin{aligned} \left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} &= g_A^4 \left|M_{GT}\right|^2 \left\{ C_1 \left(\frac{|m_{\beta\beta}|}{m_e}\right)^2 + C_2 \frac{|m_{\beta\beta}|}{m_e} \langle\lambda\rangle \cos\psi_1 + C_3 \frac{|m_{\beta\beta}|}{m_e} \langle\eta\rangle \cos\psi_2 \right. \\ &+ \left. C_4 \langle\lambda\rangle^2 + C_5 \langle\eta\rangle^2 + C_6 \langle\lambda\rangle \langle\eta\rangle \cos\left(\psi_1 - \psi_2\right) \right\} . \end{aligned} \tag{26}$$

Here $g_A = 1.269$ is the weak axial coupling constant and the effective coupling constants and their relative phases are defined as

$$\begin{aligned} \langle\lambda\rangle &= \lambda | {\sum_j}' U_{ej} V_{ej} (g_V'/g_V)|, \qquad \langle\eta\rangle = \eta | {\sum_j}' U_{ej} V_{ej}'|, \\ \psi_1 &= \arg[\{{\sum_j}' m_j U_{ej}^2\}\{{\sum_j}' U_{ej} V_{ej} (g_V'/g_V)\}], \\ \psi_2 &= \arg[\{{\sum_j}' m_j U_{ej}^2\}\{{\sum_j}' U_{ej} V_{ej}'\}^*], \end{aligned}$$

where the prime symbol over the sum means, that we sum over only lights neutrinos ($m_j \leq$ 10MeV).

The combination of nuclear matrix elements and phase-space factors can be read as

$$
\begin{aligned}
C_1 &= (1-\chi_F+\chi_T)^2 G_{01}, \\
C_2 &= -(1-\chi_F+\chi_T)\left[\chi_{2-}G_{03}-\chi_{1+}G_{04}\right], \\
C_3 &= (1-\chi_F+\chi_T)\left[\chi_{2+}G_{03}-\chi_{1-}G_{04}-\chi_P G_{05}+\chi_R G_{06}\right], \\
C_4 &= \chi_{2-}^2 G_{02}+\frac{1}{9}\chi_{1+}^2 G_{011}-\frac{2}{9}\chi_{1+}\chi_{2-}G_{010}, \\
C_5 &= \chi_{2+}^2 G_{02}+\frac{1}{9}\chi_{1-}^2 G_{011}-\frac{2}{9}\chi_{1-}\chi_{2+}G_{010}+\chi_P^2 G_{08}-\chi_P\chi_R G_{07}+\chi_R^2 G_{09}, \\
C_6 &= -2[\chi_{2-}\chi_{2+}G_{02}-\frac{1}{9}\left(\chi_{1+}\chi_{2+}+\chi_{2-}\chi_{1-}\right)G_{010}+\frac{1}{9}\chi_{1+}\chi_{1-}G_{011}]. \qquad (27)
\end{aligned}
$$

The nuclear matrix elements M_{GT} and χ_I are defined in thesis. The integrated kinematical factors are defined as

$$
\begin{aligned}
G_{0k} &= \frac{G_\beta^4 m_e^2}{32\pi^5 \ln 2 R^2}\int \delta(\varepsilon_1+\varepsilon_2+M_f-M_i)g_{0k}(\varepsilon_1,\varepsilon_2,R)p_1p_2\varepsilon_1\varepsilon_2 d\varepsilon_1 d\varepsilon_2 \\
& \qquad k=1,2,\cdots,11. \qquad (28)
\end{aligned}
$$

Here, p_1 and p_2 are momenta of electrons. θ is the angle between emitted electrons. The functions $g_{0k}(\varepsilon_1,\varepsilon_2,R)$ in which enter the $s_{1/2}$ and $p_{1/2}$ electron wave functions at nuclear surface associated with emission of both electrons are defined in thesis. These definitions are free of the weak axial-vector coupling constant g_A. The quantities G_{0k} are given in units of inverse years. We note that if the standard wave functions of electron (w.f. A) are assumed $G_{010}=G_{03}$ and $G_{011}=G_{04}$. If in addition contributions from the induced pseudoscalar term of nucleon current are neglected the decay rate in Eq. (26) reduces to that given in [11].

5.2 Phase-space factors with improved accuracy

A numerical computation of all 11 phase-space factors entering to the $0\nu\beta\beta$-decay rate was performed by using 4 types of wave functions (A, B, C and D) for a sample of 3 isotopes (^{76}Ge, ^{130}Te and ^{150}Nd). Results are presented in Table 3. We see that by using standard treatment of electron wave functions corresponding to leading finite-size Coulomb (wave functions A) a significant difference with results other three approaches appear especially for nuclei with large Z number. Surprisingly, results obtained with wave functions B corresponding to an analytical solution of Dirac equations for a pointlike nucleus better agree with results corresponding to wave functions C and D (exact solution of Dirac equations for a uniform charge distribution in nucleus at r=R) as those obtained by the standard treatment of wave functions (wave functions A). It indicates that Coulomb corrections play more important role as the position of decaying nucleon in the nucleus. By glancing the Table 3 we see that effect of the screening of atomic electrons on the wave functions of emitted electrons does not play an important role.

5.3 Constraints on the effective neutrino mass and the coupling strengths of the right-handed currents.

The $0\nu\beta\beta$-decay half-life (26) allows to discuss constraints on the effective Majorana neutrino mass $m_{\beta\beta}$, the effective coupling constants $\langle\lambda\rangle$ and $\langle\eta\rangle$ of the right-handed leptonic current with the right- and left-handed hadronic currents from experimental half-life limits, if values of phase-space factors and nuclear matrix elements are available. We shall exploit the

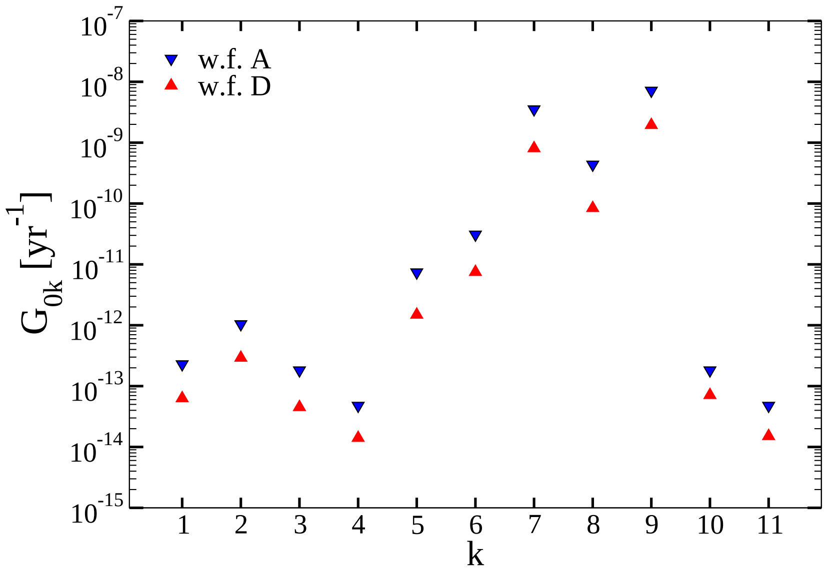

Figure 7: The phase factors G_{0k} (k=1, $\cdots$, 11) in units of yr^{-1} for the $0\nu\beta\beta$-decay of ^{150}Nd. Results are presented for approximate electron wave functions (type A [11]) and exact Dirac wave functions with finite nuclear size and electron screening (type D [12]).

Table 3: Phase-space factors G_{0k} (k=0, $\cdots$,11) in units yr^{-1} for the $0\nu\beta\beta$-decay of ^{76}Ge,^{130}Te, and ^{150}Nd. Calculation was performed by assuming different approximations for the radial wave functions $g_{\pm1}$ and $f_{\pm1}$ of an electron: A) The standard approximation of Doi et al [11]; B) An analytical solution of Dirac equations for a pointlike nucleus is assumed; C) An exact solution of Dirac equations for a uniform charge distribution in nucleus is considered; D) The same as the previous case but the electron screening is taken into account [12].

	^{76}Ge				^{130}Te				^{150}Nd			
	A	B	C	D	A	B	C	D	A	B	C	D
$G_{01}.10^{14}$	0.26	0.24	0.24	0.24	1.81	1.54	1.45	1.43	8.83	6.99	6.43	6.32
$G_{02}.10^{14}$	0.43	0.40	0.40	0.39	4.68	4.06	3.85	3.76	40.19	32.40	29.87	29.19
$G_{03}.10^{15}$	1.48	1.34	1.32	1.31	12.24	9.57	9.07	8.97	70.03	49.47	45.59	45.13
$G_{04}.10^{15}$	0.50	0.49	0.48	0.47	3.63	3.32	3.09	3.02	18.34	16.00	14.35	14.07
$G_{05}.10^{13}$	0.79	0.73	0.57	0.57	6.39	5.19	3.84	3.79	28.54	21.18	15.06	14.87
$G_{06}.10^{12}$	0.61	0.55	0.54	0.53	3.09	2.40	2.26	2.23	11.92	8.32	7.59	7.50
$G_{07}.10^{10}$	0.37	0.35	0.27	0.27	2.71	2.38	1.79	1.76	13.63	11.36	8.23	8.09
$G_{08}.10^{11}$	0.25	0.24	0.15	0.15	2.88	2.65	1.58	1.55	16.83	15.00	8.56	8.41
$G_{09}.10^{10}$	1.36	1.26	1.24	1.22	6.40	5.35	5.06	4.97	27.58	21.53	19.80	19.45
$G_{010}.10^{14}$	0.15	0.15	0.14	0.14	1.22	1.46	1.16	1.14	7.00	10.54	7.23	7.11
$G_{011}.10^{15}$	0.50	0.50	0.48	0.48	3.63	3.56	3.22	3.15	18.34	18.33	15.38	15.06

quasiparticle random phase approximation (QRPA) [13] and interacting shell-model (ISM) [14, 15], matrix elements for such an analysis.

The constraints on the effective right-handed current couplings $\langle\lambda\rangle$, $\langle\eta\rangle$ and the effective Majorana neutrino mass $m_{\beta\beta}$ are listed in Table 4. Fig. 8 shows the allowed regions for $m_{\beta\beta}$ and $\langle\lambda\rangle$ ($\langle\eta\rangle$) for $\langle\eta\rangle = 0$ ($\langle\lambda\rangle = 0$). Results are presented for two sets of nuclear matrix elements (ISM [14, 15] and QRPA [13]) and the standard (w.f. A) and improved (w.f. D) description of electron wave functions. We notice that limits on $m_{\beta\beta}$ are softened a little when $\langle\lambda\rangle$ or $\langle\eta\rangle$ have non-vanishing values at the same time in comparison with the case

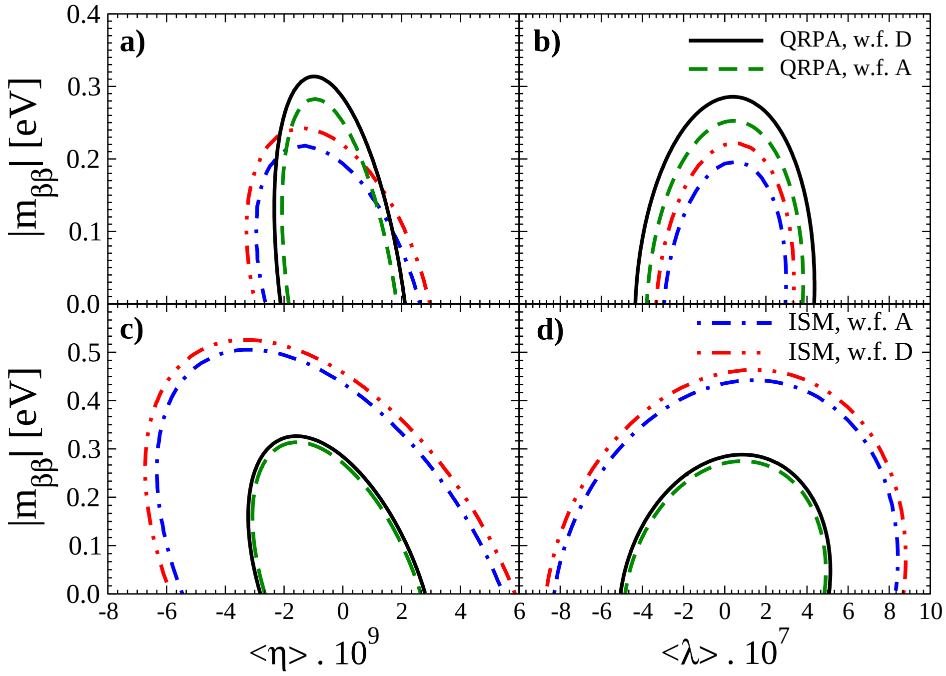

Figure 8: Limits on the effective neutrino mass $m_{\beta\beta}$ and right handed parameters η (left panels, $\langle\lambda\rangle = 0$) and λ (right panels, $\langle\eta\rangle = 0$) imposed by the constraints on the $0\nu\beta\beta$-decay of ^{76}Ge (upper panels, $T^{0\nu}_{1/2} \geq 3.0 \times 10^{25}$ [16]) and ^{136}Xe (lower panels, $T^{0\nu}_{1/2} \geq 3.4 \times 10^{25}$ [17]). To derive the bounds, the values of nuclear matrix elements calculated within the ISM [14] and the QRPA [13] are used. Results are presented for approximate electron wave functions (type A) and exact Dirac wave functions with finite nuclear size and electron screening (type D). Ellipsoids show the boundaries of the allowed domains.

Table 4: Upper bounds on the effective neutrino mass $m_{\beta\beta}$ and parameters $\langle\eta\rangle$ and $\langle\lambda\rangle$ associated with right-handed currents mechanisms imposed by the constraints on the $0\nu\beta\beta$-decay of ^{76}Ge (upper panels, $T^{0\nu}_{1/2} \geq 3.0 \times 10^{25}$ [16]) and ^{136}Xe ($T^{0\nu}_{1/2} \geq 3.4 \times 10^{25}$ [17]). Nuclear matrix elements of interactring shell-model (ISM) [14] (M_{GT} is from [15]) and quasiparticle random phase approximations (QRPA) [13] are used in analysis. CP conservation is assumed ($\psi_1 = \psi_2 = 0$). The standard electron wave functions (w.f. A) [11] and screened exact finite-size Coulomb wave functions (w.f. D) are considered.

	^{76}Ge		^{136}Xe	
w.f.	A	D	A	D
		QRPA		
$\lvert m_{\beta\beta}\rvert$ [eV]	0.321	0.333	0.285	0.315
$\lvert m_{\beta\beta}\rvert$ [eV] (for $\langle\eta\rangle = \langle\eta\rangle = 0$)	0.271	0.284	0.251	0.285
$\langle\eta\rangle \times 10^{-9}$	3.093	3.239	2.077	2.337
$\langle\lambda\rangle \times 10^{-7}$	4.943	5.163	3.822	4.370
		ISM		
$\lvert m_{\beta\beta}\rvert$ [eV]	0.515	0.535	0.222	0.245
$\lvert m_{\beta\beta}\rvert$ [eV] (for $\langle\eta\rangle = \langle\eta\rangle = 0$)	0.436	0.458	0.193	0.220
$\langle\eta\rangle \times 10^{-9}$	6.370	6.760	2.975	3.291
$\langle\lambda\rangle \times 10^{-7}$	8.462	8.841	3.000	3.378

when both these parameters are equal to zero.

5.4 Conclusions

We rederived the decay rate for the neutrinoless double-beta decay assuming the Majorana neutrino mass mechanism with the inclusion of the right-handed leptonic and hadronic currents. $p_{1/2}$-states of emitted electrons and recoil corrections to nucleon currents were taken into account. Within a standard approximation the decay rate was factorized as a sum of products of kinematical phase-space factors and nuclear matrix elements. Unlike in derivation of [11] the induced pseudoscalar term of hadron current was considered what results in additional nuclear matrix elements. The phase-space factors were expressed with Dirac component of radial wave functions of electron in the $s_{1/2}$ and $p_{1/2}$ wave states. An improved numerical computation of the phase-space factors based on exact Dirac wave functions with finite nuclear size and electron screening was presented. The dependence of values of phase-space factors on the different approximation schemes used in evaluation of electron wave functions was discussed. It was shown that Coulomb corrections play more important role as finite nuclear size effect. The upper limits for effective neutrino mass and right-handed current mechanisms were deduced from data on the $0\nu\beta\beta$-decay of ^{76}Ge and ^{136}Xe. For that purpose nuclear matrix elements calculated within the nuclear shell model and quasiparticle random phase approximation were exploited. The subject of interest was also differential decay rates for various combinations of the total lepton number violating parameters.

6 Summary

The aims of thesis have been achieved and results presented at conferences and published in several international journals. The theoretical results obtained within this PhD thesis are expected to be important for the scientific community working on the double beta decay theory and experiments, and nuclear structure theory.

7 List of publications

I. D. Štefánik, F. Šimkovic, K. Muto and A. Faessler:
Two-neutrino double-β decay Fermi transition and two-nucleon interaction.
Phys. Rev. C 88, 025503 (2013)

II. D. Štefánik, F. Šimkovic:
Double Fermi matrix element within perturbation theory.
Rom. Journ. Phys., Vol. 58, No. 9-10, P. 1251–1257 (2013)

III. D. Štefánik, F. Šimkovic and A. Faessler:
Energy-weighted sum rules connecting $\Delta Z = 2$ nuclei within the SO(8) model.
Workshop on Calculation of Double-Beta-Decay Matrix Elements (MEDEX '13), AIP Conf. Proc., 98 (2013)

IV. D. Štefánik, R. Dvornický and F. Šimkovic:
Neutrinoless double beta decay with emission of s and p electrons.
Nuclear Theory, Vol. 33, 115 (2014)

V. D. Štefánik, F. Šimkovic and A. Faessler:
Structure of the two-neutrino double-beta decay matrix elements within perturbation theory.
Accepted in Phys. Rev. C (2015)

VI. D. Štefánik, R.Dvornický, F. Šimkovic and P. Vogel:
Phase-space factors for the neutrinoless double beta decay with the right-handed currents.
To be submitted in Phys. Rev. C (2015)

8 Contributions in the conferences, workshops and seminars

I. "Two-neutrino double beta decay matrix elements within SO(5) and SO(8) models (50 min)", Seminar on nuclear theory, BLTP JINR Dubna, Russia, 8. April, 2013.

II. "Two-neutrino double beta decay matrix elements within SO(5) and SO(8) models (25 min)", MEDEX'13, Prague, Czech Republic, June 11 - 14, 2013.

III. "$2\nu\beta\beta$-decay within SO(5) and SO(8) models and energy-weighted sum rule involving $\Delta Z = 2$ nuclei (35 min)", International School of Nuclear Physics, 35th Course - Neutrino Physics: Present and Future, Erice-Sicily, Italy, September 16-24, 2013 .

IV. Poster session on the WE-Heraeus-Seminar on 'Massive Neutrinos', Bad Honnef, Germany, April 21-25, 2014. "Two-neutrino double-β decay Gamow-Teller transition and two nucleon residual interactions";

V. "Neutrinoless double beta decay with Majoron emission revisited (15 min)", IWNT33-2014, Rila Mountains, Bulgaria, June 22-28, 2014.

VI. "Neutrinoless double beta decay with emission of s and p electrons (10 min)", AYSS-2015, Dubna, Russia, February 16-20, 2015.

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