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Mathematical Analysis and Calibration of a Multifactor Panel Model for Credit Spreads and Risk-free Interest Rate

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1 Introduction

All investors in the financial market have their own risk profiles and try to find investment opportunities that best suit their risk appetite. Many investors are traditionally conservative, like insurance companies or pension funds, and seek investments with minimum credit risk. Also many other economic or financial models use the risk-free rate as a benchmark to discount future cash-flows. But what really can be considered to be the risk-free interest rate?

Usually government bonds issued by the domestic government or by other high rated governments are considered to be credit-risk-free investments. Recently also bonds issued by high rated globally active financial or non-financial corporations, interbank rates or swap rates are gaining that status. However events like the collapse of Lehman Brothers, an AAA rated investment bank, on September 15, 2008, the downgrade of the United States by Standard & Poor's on August 5, 2011 [16], or the current sovereign crisis of the euro area with the downgrade of nine euro area sovereigns by Standard & Poor's on January 13, 2012 [15] including the AAA rated France and Austria and with the default of Greece on March 8, 2012 [6] remind us once again that credit risk is related with every investment involving another counterparty, no matter how negligible it might be viewed in advance.

This applies, of course, also to sovereign investments although investors tend to forget the lessons learned in the past as history is full of sovereign defaults or debt restructuring where investors lost a significant part of their investment. According to Reinhart, Rogof and Savastano [14] there were 33 external sovereign defaults in Europe in nineteenth century and 35 defaults among the Middle Income Countries between 1970 and 2001. This number rises dramatically if internal sovereign defaults are considered as well [13].

Many prime international financial institutions including Lehman Brothers, AIG, Dexia were nationalized, sold, liquidated or acquired state aid. So the sovereign acts as the backstop for the financial markets and troubles in the financial system can lead to the sovereign crisis as demonstrated by Iceland or Ireland.

But still not enough effort is put into measuring sovereign credit risk. This is partially justified by the difficulties in splitting the sovereign yield into the risk-free part and the credit spread in an environment when the sovereign bond is the least risky asset in the market. However the creation of the euro area in 1999 provides a unique opportunity to this kind of studies. At the time this thesis is written seventeen countries in the euro area form a currency union using the single European currency, the Euro. This forms an environment where government bonds with different levels of credit risk are issued, while having all other properties virtually the same. Using this broad range of data it should be possible to define the common risk-free rate for the euro area and the unique credit spreads for every country.

This opportunity was explored by Geyer, Kossmeier and Pichler [4] and Puig [12], who studied the credit spreads of the euro area countries over the German yield curve. However the German yield was considered to be the risk-free rate in both approaches. Kaminska, Meldrum and Smith [7] studied a model involving interest rates of the United States, United Kingdom and German yields and exchange rates between the US dollar, British pound and the Euro. Monfort and Renne [11] studied the default and liquidity risks of ten euro area countries using the no-arbitrage framework. But in all these traditional approaches some of the market instruments, which is believed to be the least risky is chosen as the risk-free rate.

2 Goals of the thesis

The main aims of this thesis are:

- develop a new multifactor panel model for the risk-free rate and credit spreads of the euro area sovereigns and price the sovereign bonds using this model
- develop the calibration method to estimate model parameters
- estimate the true unobserved risk-free rate and credit spreads of the euro area countries using the sovereign yield curves for the whole period of the euro area existence
- compare the development of the risk-free rate in different periods of time
- discuss the results

3 Results

3.1 Initial model specification (CIR)

The idea, inspired by Leško [10], is based on the multifactor CIR model [3] which we broadened to a multiple equation model for multiple countries. We assume that the short-rate for every sovereign is a sum of the risk-free rate common to the whole euro area and a unique credit spread for every country. Both the risk-free rate and credit spreads are assumed to follow a one-factor CIR process and they area uncorrelated. The model is described by:

$$
\begin{aligned}
r_t^i &= rf_t + cs_t^i, \\
drf_t &= \kappa(\theta - rf_t)dt + \sigma\sqrt{rf_t}\,dW_t, \qquad \text{for } i = 1\ldots n, \\
dcs_t^i &= \kappa_i(\theta_i - cs_t^i)dt + \sigma_i\sqrt{cs_t^i}\,dW_t^i,
\end{aligned}
$$

where W_t and W_t^i are standard Wiener processes with uncorrelated increments, $i = 1\ldots n$ represents individual Member States of the euro area and t is time. Here coefficients κ, θ and σ are the mean-reversion spread, long-term equilibrium rate and volatility, respectively, for the risk-free rate. The coefficients κ_i, θ_i and σ_i have the same interpretation for the credit spread of country i. This notation is used throughout the whole thesis.

The price of a zero-coupon sovereign bond is given by the following separated form formula (c.f. Kwok [8] or Ševčovič, Stehlíková, Mikula [19]:

$$
\begin{aligned}
P^i(t,\tau,r_t^i) &= P_{rf}^i(t,\tau,rf_t)P_{cs}^i(t,\tau,cs_t^i) = \\
&= A(\tau)e^{-B(\tau)rf_t - B_i(\tau)cs_t^i} = A(\tau)A_i(\tau)e^{-B(\tau)rf_t - B_i(\tau)cs_t^i},
\end{aligned}
$$

where $P_{rf}^i(t,\tau,rf_t)$ *and* $P_{cs}^i(t,\tau,cs_t^i)$ *are solutions to the one-factor CIR models for* rf_t *and* cs_t^i *in the form of:*

$$
\begin{aligned}
P_{rf}^i(t,\tau,rf_t) &= A(\tau)e^{-B(\tau)rf_t}, P_{cs}^i(t,\tau,cs_t^i) = A_i(\tau)e^{-B_i(\tau)cs_t^i}, \\
A(\tau) &= \left[\frac{\eta e^{(\kappa+\bar{\lambda}\sigma+\eta)\tau/2}}{e^{\eta\tau}-1}B(\tau)\right]^{2\kappa\theta/\sigma^2}, A_i(\tau) = \left[\frac{\eta_i e^{(\kappa_i+\bar{\lambda}_i\sigma_i+\eta_i)\tau/2}}{e^{\eta_i\tau}-1}B_i(\tau)\right]^{2\kappa_i\theta_i/\sigma_i^2}, \\
B(\tau) &= \frac{2[e^{\eta\tau}-1]}{(\kappa+\bar{\lambda}\sigma+\eta)[e^{\eta\tau}-1]+2\eta}, B_i(\tau) = \frac{2[e^{\eta_i\tau}-1]}{(\kappa_i+\bar{\lambda}_i\sigma_i+\eta_i)[e^{\eta_i\tau}-1]+2\eta_i}, \\
\eta &= \sqrt{(\kappa+\bar{\lambda}\sigma)^2+2\sigma^2}, \eta_i = \sqrt{(\kappa_i+\bar{\lambda}_i\sigma_i)^2+2\sigma_i^2}.
\end{aligned}
$$

The yield of a zero coupon bond of country i is an affine function of the risk-free rate and credit spread given by the formula:

$$R^i(\tau) = -\frac{\ln(A(\tau)) + \ln(A_i(\tau)) - B(\tau) rf_t - B_i(\tau) cs_t^i}{\tau}.$$

3.2 Final model specification (CKLS)

The initial model specification was chosen after the survey of interest rate models in Chapter 2 of the thesis. The CIR model was chosen for its nonnegativity of interest rates, time-varying volatility and the convenient analytical formula for the price of a zero-coupon bond. After we estimated the initial model we found out that the assumption of zero correlation between the Wiener processes of the risk-free rate and credit spreads is non-realistic for the data we used. We found out that they are significantly negatively correlated, although we can find no economic reasoning why these two variables should be dependent. However in such a case no analytical formula for the price of a zero coupon bond exists. As we are forced to use an approximate bond price formula, we decided also to extend the model specification to the more general Chan, Karolyi, Longstaff, Sanders model [2]:

$$\begin{aligned} r_t^i &= rf_t + cs_t^i, \\ drf_t &= \kappa\left(\theta - rf_t\right) dt + \sigma rf_t^{\gamma} dW_t, \\ drf_t &= \kappa\left(\theta - rf_t\right) dt + \sigma rf_t^{\gamma} dW_t, \\ Cov\left(dW_t, dW_t^i\right) &= \rho_i dt, \end{aligned} \quad \text{, for } i = 1 \ldots n,$$

where W_t and W_t^i are standard Wiener processes.

In this case no analytical formula exists for the price of a zero-coupon bond. We chose the approximate price of a zero-coupon bond recently developed by Halgašová [5] for the two factor CKLS model. The idea behind the approximate formula is that for a two-factor Vasicek model there exists an analytical formula for the bond price even in the case of correlated Wiener processes. The approximate formula for the CKLS model is then given by substituting the CKLS volatility term instead of σ in the analytical formula for the Vasicek model. The approximate formula is defined by:

$$P_i^{ap}(t,\tau,r_1,r_2) = a_i^{ap}(\tau) e^{-B(\tau) rf_t - B_i(\tau) cs_t^i},$$

$$\ln a_i^{ap}(\tau) = \ln A^{ap}(\tau) + \ln A_i^{ap}(\tau) + \rho_i \frac{\sigma \sigma_i rf_t^{\gamma} \left(cs_t^i\right)^{\gamma_2}}{\kappa \kappa_i} \left[\tau - B(\tau) - B_i(\tau) + \frac{1 - e^{(\kappa+\kappa_i)\tau}}{\kappa + \kappa_i} \right],$$

$$\ln A^{ap}(\tau) = \left(B(\tau) - \tau\right) \left[\theta - \frac{\sigma^2 rf_t^{2\gamma}}{2\kappa^2} - \frac{\sigma \lambda rf_t^{\gamma}}{2\kappa} \right] + \frac{\sigma^2 rf_t^{2\gamma}}{4\kappa} B^2(\tau), \qquad .$$

$$\ln A_i^{ap}(\tau) = \left(B_i(\tau) - \tau\right) \left[\theta_i - \frac{\sigma_i^2 \left(cs_t^i\right)^{2\gamma_2}}{2\kappa_i^2} - \frac{\sigma_i \lambda_i \left(cs_t^i\right)^{\gamma_2}}{2\kappa_i} \right] + \frac{\sigma_i^2 \left(cs_t^i\right)^{2\gamma_2}}{4\kappa_i} B_i^2(\tau),$$

$$B(\tau) = \frac{1 - e^{\kappa\tau}}{\kappa}, B_i(\tau) = \frac{1 - e^{\kappa_i \tau}}{\kappa_i}.$$

The yield of a zero coupon bond of country i is not an affine function of the risk-free rate and credit spread anymore and is given by the formula:

$$R^i(\tau) = -\frac{\ln(a(\tau, rf_t, cs_t^i)) - B(\tau) rf_t - B_i(\tau) cs_t^i}{\tau}.$$

3.3 Calibration methodology

The initial CIR model was calibrated using the extended version of the minmax method developed by Ševčovič and Urbánová Csajková [18] and Urbánová Csajková [20]. The methodology is based on minimizing the cost function defined by:

$$U\left(\boldsymbol{\kappa}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\lambda}, \boldsymbol{r}\right) = \sum_{t=1}^{N} \sum_{j=1}^{m} \sum_{i=1}^{n} \left(\tilde{R}_t^i(\tau_j) - R_t^i(\tau_j) \right)^2,$$

where $\tilde{R}_t^i(\tau_j)$ is the real yield of a zero-coupon government bond of country i with residual maturity τ_j observed at time t, that is the sum of residuals through every country i, every maturity τ_j and every time observation t. Note that the value of the cost function depends not only on the parameters of the CIR processes for the risk-free rate and credit spreads, but also on the actual development of these processes, because they are unobserved variables.

The parameters κ or λ and κ_i or λ_i are free parameters in the model as they appear in the bond price formula only in the terms $\kappa + \lambda\sigma$ and $\kappa_i + \lambda_i \sigma_i$. The minimization is therefore done in the transformed variables $\psi = (\phi, \phi_1, \ldots, \phi_N) = (\beta, \xi, \rho, \ldots \beta_n, \xi_n, \rho_n)$ where the transformation for the one-factor model was developed by Ševčovič and Urbánová Csajková [18].

The price of a zero-coupon bond is defined on an open space $\psi \in (0,1)^{2n+2} \times (0,\infty)^{n+1} \subset \mathbb{R}^{3n+3}$. However as the parameters tend to their boundaries, the original parameters tend to zero or infinity, which is a degenerate solution. Therefore we will search for the estimate of the parameters ψ on the following compact subspace:

$$B = \left\{ \psi \in \mathbb{R}^{3n+3} \left| \begin{matrix} \delta \le \beta \le 1-\delta, \delta \le \beta_i \le 1-\delta \\ \delta \le \xi \le 1-\delta, \delta \le \xi_i \le 1-\delta \\ \delta \le \tilde{\rho} \le \tilde{\rho}_{\max}, \delta \le \tilde{\rho}_i \le \tilde{\rho}_{\max} \end{matrix} \right. \right\},$$

where δ is small enough and $\tilde{\rho}_{\max}$ is large enough. See Section 5.1 of the thesis for more details on the transformation.

Let us denote $\boldsymbol{r}_t = \left\{ \left(rf_t, cs_t^1, \ldots, cs_t^n \right), t > 0 \right\}$. If $\psi \in B$ then $U(\psi, \boldsymbol{r}) \to \infty$ if $rf_t \to \infty$ or $cs_t^i \to \infty$ for any i. Therefore the minimum of $U(\psi, \boldsymbol{r})$ is attained on a space $S_C = \left\{ \boldsymbol{r} \ge 0, \sum_{i=0}^{n} \boldsymbol{r}_i \le C \right\} \subset \mathbb{R}^{n+1}$ where C is a sufficiently large constant.

We search for the solution to the following minimization problem:

$$\min_{\psi, \boldsymbol{r}} U(\psi, \boldsymbol{r}),$$
$$\psi \in B, \boldsymbol{r} \in S_C.$$

Due to the size of the optimization problem this is done in a two step approach:

$$\min_{\psi, \boldsymbol{r}} U(\psi, \boldsymbol{r}) = \min_{\psi} \min_{\boldsymbol{r}} U(\psi, \boldsymbol{r})$$

We managed to prove the following two propositions:

Proposition 25. *For every fixed value ψ the inner minimization problem*

$$\min_{\boldsymbol{r}} U(\psi, \boldsymbol{r})$$
$$\boldsymbol{r} \ge 0$$

has a unique global minimum. Let us denote this unique global minimum $\boldsymbol{r}(\psi) = \arg\min_{\boldsymbol{r} \in S_C} U(\psi, \boldsymbol{r})$.

Proposition 26. *The functions* $\psi \to \boldsymbol{r}(\psi)$ *as well as* $\psi \to U\left(\psi, \boldsymbol{r}(\psi)\right)$ *are continuous.*

For a fixed value of the parameter vector ψ the yield to maturity function is an affine function in the variables $rf(\cdot)$ and $cs^i(\cdot)$. Thus the inner problem is a ordinary least squares estimate with constraints $\boldsymbol{r}_t \geq \boldsymbol{0}$ for every t.

From a numerical point of view it is a quadratic programming problem, which can be efficiently and reliably solved. Let us denote $\boldsymbol{r}(\psi)$ the obtained unique minimum of the inner problem for a fixed parameter value ψ. In practice we used the Matlab function *lsqnonneg* to solve the inner problem, which uses the algorithm described in Lawson and Hanson [9, Chapter 23].

Following from Proposition 26, the outer problem does have a minimum in B, although it may not be unique. As the outer problem need not be a good function to optimize the simulated annealing method is then used, to find the global minimum of the outer problem:

$$\min_{\psi} U\left(\psi, \boldsymbol{r}(\psi)\right).$$

The outer problem was solved using the Matlab function *simulannealbnd* from the Genetic algorithm and Direct Search Toolbox. We used $\delta = 10^{-12}$ and $\tilde{\rho}_{\max} = 250$.

Finally in order to estimate the original parameters of the CIR processes we find the maximum of the likelihood function of the discretized CIR model, which according to Bergstrom [1] has the following form:

$$\ln L(\kappa, \theta, \sigma) = -\frac{1}{2} \sum_{t=2}^{N} \left(\ln v_t^2 + \frac{\varepsilon_t^2}{v_t^2} \right)$$

where $v_{t+\Delta t}^2 = \dfrac{\sigma^2}{2\kappa}(1 - e^{-2\kappa\Delta t}) r_t$, $\varepsilon_{t+\Delta t} = r_{t+\Delta t} - e^{-\kappa\Delta t} r_t - \theta(1 - e^{-\kappa\Delta t})$ for rf_t as well as for every cs_t^i.

However, this approach cannot be used for the final model specification. Although in Ševčovič and Urbánová Csajková [18] a parameter transformation was suggested also for the Vasicek model, it cannot be applied to the approximate bond price because the terms A^{ap}, A_i^{ap}

involve rf_t and cs_t^i. It is also not necessary to introduce the transformation as no parameter is free in the approximate bond price formula. Therefore we will calibrate the model directly in the original parameters including correlations between the risk-free rate and credit spreads and the powers $\tilde{\psi} = (\tilde{\phi}, \tilde{\phi}_1, \ldots, \tilde{\phi}_N, \boldsymbol{\rho}, \gamma, \gamma_2) = (\kappa, \theta, \sigma, \lambda, \ldots, \kappa_n, \theta_n, \sigma_n, \lambda_n, \rho_1, \ldots \rho_n, \gamma, \gamma_2)$.

Another drawback is that the inner minimization problem is no longer an affine function in $rf(\cdot)$ and $cs^i(\cdot)$ and its solving is much more time intensive.

Therefore we adopted the following approach. As the estimate of the factors $\boldsymbol{r}$ in the CIR model is quite robust, we consider this estimate to be sufficient and we solve only the outer problem:

$$\begin{gathered} \min_{\tilde{\psi}} U\left(\tilde{\psi}, \boldsymbol{r}(\hat{\psi})\right) \\ \kappa, \kappa_i, \theta, \theta_i, \sigma, \sigma_i, \gamma, \gamma_i \geq \delta \,. \\ -1 \leq \rho_i \leq 1 \end{gathered}$$

where $\hat{\psi}$ is the optimal solution to the CIR model with zero correlation.

The outer problem was solved in the original CKLS parameter space using again simulated annealing in Matlab with $\delta = 10^{-12}$. For the optimal value of the parameters the inner problem and again the outer problem was solved. The results did not differ significantly from $\boldsymbol{r}(\hat{\psi})$.

3.4 Calibration results

We managed to calibrate the initial and final model specification. The estimate of the risk-free rate and credit spreads proved to be quite robust and in line with economic expectations both using the CIR and CKLS models and also if estimated using different time periods. This justifies our approach to the final model calibration. The major result from the estimation is that the risk-free rate drops below zero in the deep financial crises periods like the one we are facing now. This is supported by the fact, that in the crisis period the power for the risk-free rate in the CKLS model is close to zero suggesting the Vasicek specification of the process which allows negative interest rates.

The model was not able to capture the significant increases in yield for Ireland and Portugal, which would default without coordinated international support and for Greece, which defaulted despite the coordinated international support. However the model performed quite well if estimated without these three countries.

The parameters estimated for the initial model specification were satisfactory only for the risk-free rate where they correspond also to the estimates using the CKLS model specification. However the significant negative correlation between the estimated risk-free rate and credit spreads was observed, ranging from -0.3 to -0.7 depending on the period used. For the credit spreads the estimated parameters in the CIR specification were very volatile and even negative in some cases and unrelated to the actual development of the credit spreads.

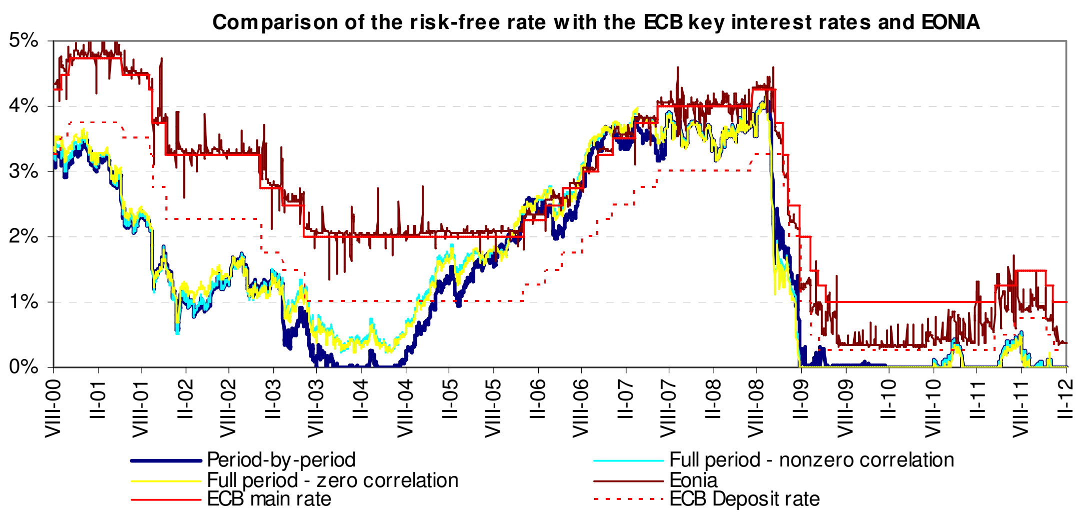

Figure 1 Comparison of the estimated risk-free rate with the ECB Key Interest Rates and the EONIA. The risk-free rate estimated period-by-period and for the full period with and without zero correlation. The ECB main rate is the minimum bid rate for the variable rate tenders up to October 15, 2009 and the fixed rate for tenders since then. Source: ECB[1], Thomson-Reuters, Own calculations.

Therefore we decided to calibrate the CKLS version of the model with correlation between the risk-free rate and credit spreads. Again, the model was not able to capture the developments for Greece, Portugal and Ireland, however without these countries the results were good and slightly better than in the case of the CIR noncorelation model. The processes are mainly volatility driven with the mean-reversion speed less significant even for small values of the risk-free rate and credit spreads. During the recent financial crisis for the least

[1] www.ecb.int

risky countries both the risk-free rate and credit spreads are zero and this is still not enough to capture the unnaturally yields of these countries. The order of countries is as expected with Italy and Spain having the highest long-term equilibrium spreads while Germany and France having the lowest.

Table 1 Parameter estimates for the final model specification for the risk-free rate and credit spreads of euro area sovereigns for the full period without three month maturity calibrated without Greece, Portugal and Ireland.

Country	κ, κ_i	θ, θ_i	σ, σ_i	λ, λ_i	1 ρ, ρ_i	2 ρ, ρ_i	3 ρ, ρ_i	Average error
Risk-free rate	0,3060	0,0266	0,1084	0,0109	NA	NA	NA	NA
Austria	0,0421	0,0547	0,0163	-0,4688	-0,335	0,000	-0,15	13,30
Belgium	0,0500	0,0724	0,0100	0,5064	-0,397	-0,007	-0,35	14,09
Finland	0,0312	0,0599	0,0997	-0,1240	-0,248	0,001	-0,20	13,63
France	0,1153	0,0105	0,0530	-0,4374	-0,309	0,000	-0,45	17,14
Germany	0,0628	0,0238	0,1089	-0,2027	-0,383	0,010	-0,37	15,14
Italy	0,0299	0,0928	0,0986	-0,0820	-0,384	0,007	-0,43	15,38
Netherlands	0,0564	0,0362	0,0794	-0,2322	-0,344	0,000	-0,20	11,73
Spain	0,0233	0,0991	0,1312	-0,1019	-0,385	-0,002	-0,36	16,34

Estimates of the parameters $\kappa, \kappa_i, \theta, \theta_i, \sigma, \sigma_i, \lambda, \lambda_i, \rho_i$ of the final model specification for the processes of the risk-free rate and credit spreads of the euro area countries without GR, PT, and IE for full time period without three month maturity. The average error is in basis points. Correlation coefficients - estimated 1), calculated 2) and calculated without zero observations 3).

Finally the model was estimated on sub-periods which were chosen as turnarounds in the economic cycle. For all sub-periods and for the total model the CIR model specification is justified with the estimate ranging from 0.45 to 0.62. The only exception is the period after the collapse of Lehman Brothers, were the power for the risk-free rate was 0.12 (close to the Vasicek model) and 1 for the credit spreads. The parameter estimates for the risk-free rate are robust, but the credit spread estimates are more volatile and in some cases showing too high long-term equilibrium spreads which cannot be justified. This might be caused by the approximate formula for the bond price where the parameters θ and λ are present only in one of the terms, which results to different estimates with very similar function values.

4 Summary

We developed a new multifactor panel model for the euro area risk-free rate and credit spreads of the euro area countries. The model assumes that the short-rate for every sovereign is a sum of the risk-free rate common to the whole euro area and a unique credit spread for every country. Both the risk-free rate and credit spreads are assumed to be unobserved variables in the market as well as the short-rate for every country. The only observable variables are yields of the sovereign bonds. Both the risk-free rate and credit spreads are modeled using the Cox, Ingersoll, Ross (CIR) and the Chan, Karolyi, Longstaff, Sanders (CKLS) processes using the recently developed approximate bond price formula for this kind of model.

The advantage of the model is that it allowed us to describe the true risk-free rate for the euro area and asses how much of the sovereign yield can be attributed to the risk-free part and how much is the compensation for the credit risk of the issuing country. We did not need to specify any existing asset as the risk-free asset. We also managed to estimate the specification of the model (the power parameter in the CKLS model) and correlations between the risk-free rate and credit spreads and to observe their changes in time.

We calibrated the model using euro area zero-coupon sovereign yield curves from August 1, 2000 to February 3, 2012 with maturities between three months and ten years. We managed to split the yield between the common risk-free rate of the euro area and the individual credit spreads of the euro area countries. The resulting general development and shape of the risk-free rate and credit spreads are robust and do not change significantly if estimated on sub-periods of data both with regard to time or group of countries used. Their shapes correspond to the economic expectations and to the ECB Key Interest rates, when in periods of economic slowdown the risk-free rate decreases, while in booming periods it increases. The credit spread behavior is opposite to the risk-free rate with quite a strong negative correlation between them. Our result suggest that in severe economic recession as the one experienced after the collapse of Lehman Brothers in September 2008 the usual assumption of nonnegative interest rates is unrealistic. Our estimates suggest that the risk-free rate should drop as low as -1.6 %. Even the credit spreads of the countries perceived to be least risky would need to drop into negative territory in order to capture the extremely low yields of these countries during the recent financial crisis. These conclusions hold also if the model is estimated only on the crisis period data.

The levels of the risk-free rate and credit spreads are not that robust as the overall shape. If the model is estimated on sub-periods of data, the general trend is that the risk-free rate and credit spread values increase for their high values compared to the full period estimate and decrease for the low values of their estimates. The model was not able to capture the development in yields of the countries which defaulted like Greece, or which needed coordinated international support like Ireland and Portugal. For the other countries the model performed quite well. The estimates were more precise for the middle part of the yield curve, less precise for the long-term part of the curve and worse for the very-short term part of the curve due to low liquidity of the short-term part of the sovereign yield curves.

Our estimates showed that the assumption of zero correlation between the risk-free rate and credit spreads is non-realistic and a strong negative correlation ranging between -0.3 and -0.7 is present in the data. However the CIR specification of the model can be confirmed except the immediate period after the collapse of Lehman Brothers where the risk-free rate process is closer to the Vasicek model and the credit spread volatility depends on the credit spreads with the power of 1.

Generally all processes are mainly driven by the stochastic part of the process rather than the mean reversion trend. The estimated parameters for the risk-free rate are quite robust both with regard to time periods and country groups used for the estimation. The estimated parameters for the credit spreads were quite volatile and showed in some cases unrealistically high estimates for the long-term equilibrium spreads.

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Abstract

In this thesis a new panel model is developed for the sovereign bond yields in the euro area. We assume that the short-rate of a particular euro area country is a sum of two unobserved underlying processes: the risk-free rate and a credit spread. The risk-free rate is common for all euro area countries, while the credit spread is idiosyncratic for every country. Both the risk-free rate and the credit spreads are modeled by the well-known one-factor Cox, Ingersoll, Ross or Chan, Karolyi, Longstaff, Sanders process. The model is calibrated using daily data on yield curves for the whole existence of the euro area until February 3, 2012 and the results are discussed.

Key words: panel model, multifactor model, credit spread, risk-free rate

Abstrakt

V dizertačnej práci navrhujeme nový panelový model pre výnosové krivky štátov eurozóny. Predpokladáme, že okamžitá úroková miera je súčtom dvoch nepozorovateľných procesov: bezrizikovej sadzby a kreditného spredu. Bezriziková sadzba je spoločná pre všetky krajiny eurozóny, kým kreditný spred je samostatný pre každú krajinu. Aj bezriziková úroková miera aj kreditné spredy sú modelované všeobecne známym, Cox, Ingersoll, Ross, alebo Chan, Karolyi, Longstaff, Sanders procesom. Model je kalibrovaný na denných dátach výnosových kriviek jednotlivých krajín eurozóny od začiatku jej existencie až do 3. februára 2012 a diskutujeme výsledky.

Kľúčové slová: panelový model, viacfaktorový model, kreditné spredy, bezriziková sadzba

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6.1.1 Submitted publications

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6.1.2 International Conferences

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6.1.3 Presentations

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