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**Autoreferát dizertačnej práce**

# Vznik a vývoj kozmologických perturbácií

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**4.1.2 všeobecná fyzika a matematická fyzika**

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# Zoznam publikácií[1]

1. Peter Mészáros:
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2. Peter Mészáros, Vladimír Balek:
   *Effect of radiation-like solid on small-scale CMB anisotropies*,
   Class. Quant. Grav. **34**, 175010 (2017), arXiv:1701.09061 [gr-qc].

3. Peter Mészáros:
   *Inflation driven by scalar field and solid matter*,
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[1] Uvedené publikácie neboli doteraz citované. Čiastkové alebo úplné výsledky práce vzťahujúcej sa k uvedeným publikáciám som prezentoval na nasledujúcich konferenciách:

- konferencia 'Week of Doctoral Students 2016' v Prahe (7-9 Jún 2016)
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ako aj na ďalších letných a zimných školách a seminároch.

[2] Tento článok zatiaľ nie je publikovaný. Bol odoslaný do časopisu 'International Journal of Modern Physics D' a je v recenznom procese.

# Súhrn práce

# Vznik a vývoj kozmologických perturbácií
# Creation and evolution of cosmological perturbations

## Perturbation theory in flat universe

The cosmic microwave background (CMB) anisotropies provide a very powerful tool for comparing the theory with observations. This requires an analysis within the perturbation theory. The linearised perturbation theory describes the evolution of perturbations and is sufficient for computation of the two-point correlation functions.

Since according to most of the inflationary models the vector perturbations are not generated, we may skip them. The scalar and tensor perturbations of the flat Friedmann–Robertson–Walker–Lemaître (FRWL) metric are parametrized by four scalar perturbations $\phi$, $\psi$, $B$ and $E$,

$$ds^{(\mathrm{S})2} = a^2 \left\{(1+2\phi)d\tau^2 + 2B_{,i}d\tau dx^i - \left[(1-2\psi)\delta_{ij} - 2E_{,ij}\right] dx^i dx^j\right\}, \tag{1}$$

where $\tau = \int a^{-1}dt$ is the conformal time, $a(\tau)$ is the scale factor of the universe, and the tensor perturbation are described by the tensor $\gamma_{ij}$,

$$ds^{(\mathrm{T})2} = a^2 \left[d\tau^2 - (\delta_{ij} - \gamma_{ij})\, dx^i dx^j\right], \tag{2}$$

which is transversal, ${\gamma^j}_{i,j} = 0$, and traceless, ${\gamma^i}_i = 0$ (with indices raised and lowered by the Euclidean metric).

The scalar perturbations change under the small coordinate transformation

$$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x), \tag{3}$$

so that they may vanish in one coordinate system while being nonzero in another one, which leads to problem with physical interpretation. The problem can be avoided by introducing variables which are invariant under small coordinate transformations (3). The simplest combinations of variables characterizing scalar perturbations which are invariant are

$$\Phi = \phi - \frac{1}{a}[a(B-E')]', \quad \Psi = \psi + \frac{a'}{a}(B-E'). \tag{4}$$

There are two independent invariant scalar perturbations because there are two scalar parts of $\xi^\mu$ which can be chosen appropriately to remove two of four functions $\phi$, $\psi$, $B$ and $E$. Note that $\gamma_{ij}$ is invariant.

Presence of metric perturbations usually causes formation of matter perturbations and vice versa. In order to construct a realistic model of cosmological perturbations, one has to take into account also matter perturbations. The energy density, pressure and 4-velocity perturbations which are invariant under coordinate transformation (3) are

$$\begin{aligned}
\overline{\delta\rho} &= \delta\rho - \rho^{(0)\prime}(B - E'), \\
\overline{\delta p} &= \delta p - p^{(0)\prime}(B - E'), \\
\overline{\delta u_0} &= \delta u_0 - [a(B - E')]', \\
\overline{\delta u_i} &= \delta u_i - a(B - E')_{,i}.
\end{aligned} \tag{5}$$

Dynamics of metric perturbations and perturbations of stress-energy tensor describing the energy density, pressure and 4-velocity is given by Einstein field equations,

$$G_\mu{}^\nu \equiv R_\mu{}^\nu - \frac{1}{2}R_\lambda{}^\lambda \delta_\mu^\nu = \frac{1}{2}T_\mu{}^\nu, \tag{6}$$

where $R_{\mu\nu}$ is the Ricci tensor. Due to the Bianchi identity, these equations can be supplemented by the energy-momentum conservation law, $T_\mu{}^\lambda{}_{;\lambda} = 0$.

## Cosmic microwave background

The cosmic microwave background (CMB) anisotropies are given by perturbations at the time of last scattering and effects influencing photons during their propagation to the observer. Effects influencing photons during their propagation to the observer can be described with the use of relativistic Boltzmann equation for polarization-dependent distribution function of photons. The solution for temperature fluctuation $\delta T/T$ can be expressed in the form of line-of-sight integral, and if we restrict ourselves to scalar perturbations,[3] we can write it as a sum of two terms (see §7.1 in [31]),

$$\frac{\delta T}{T}(\tau_0, \mathbf{l}) = \left(\frac{\delta T}{T}\right)_{\text{early}}(\tau_0, \mathbf{l}) + \left(\frac{\delta T}{T}\right)_{\text{ISW}}(\tau_0, \mathbf{l}), \tag{7}$$

where $\tau_0$ is conformal time today and $\mathbf{l}$ is the unit vector pointing towards the observer from the place in the sky from which the radiation is coming. The first term represents fluctuations which appeared when there was significant amount of free electrons in the universe (this includes the period of reionization, which is however not considered here), and the second term, called *integrated Sachs–Wolfe effect*, represents fluctuations which

[3]We omit the tensor perturbations, since their effect on the CMB has not been observed yet.

arise from the action of metric perturbations on radiation during its propagation to the observer.

The early part of temperature fluctuations simplifies if we assume that photons are in local equilibrium during the whole period of recombination, and that they scatter for the last time at any moment $\tau_l$ with probability distribution given by their mean free time at that moment. The line-of-sigh integral then reduces to an integral over the times $\tau_l$ of the function

$$\left(\frac{\delta T}{T}\right)_{\rm early}(\tau_0, \mathbf{l}; \tau_l) = \left(\frac{1}{4}\delta_\gamma(\tau_l, \mathbf{l}) + \Phi(\tau_l, \mathbf{l}) + \mathbf{v}(\tau_l, \mathbf{l}) \cdot \mathbf{l}\right) \mathcal{V}(\tau_l), \tag{8}$$

where $\mathcal{V}$ is the *visibility function*, $\delta_\gamma = \overline{\delta\rho_\gamma}/\rho_\gamma$ is the photon density contrast in Newtonian gauge (for simplicity, we skip the bar over $\delta$), $\mathbf{v}$ is the local velocity of the radiating matter, and $\delta_\gamma(\tau, \mathbf{l})$, $\Phi(\tau, \mathbf{l})$ and $\mathbf{v}(\tau, \mathbf{l})$ denote the values of functions $\delta_\gamma$, $\Phi$ and $\mathbf{v}$ at the conformal time $\tau$ and at the position where the photon arriving in the direction $\mathbf{l}$ was at that time. The first term in the brackets in (8) is the local contribution to $\delta T/T$ as computed from the proportionality $\rho_\gamma \propto T^4$ (Stefan–Boltzmann law), the second term, called *Sachs–Wolfe effect*, comes from the action of metric perturbations on radiation in the place of last scattering, and the third term is due to *Doppler effect*, which contributes to the observed temperature fluctuations because the radiating matter at the time of last scattering is not static.

In the terms of functions

$$\alpha(\mathbf{k}) \equiv \frac{1}{4}\delta_{\gamma\mathbf{k}} + \Phi_{\mathbf{k}}, \quad \beta(\mathbf{k}) \equiv -\frac{3}{4k^2}\left(\delta_{\gamma\mathbf{k}} - 4\Psi_{\mathbf{k}}\right)', \quad \gamma(\tau, \mathbf{k}) \equiv \Phi'_{\mathbf{k}} + \Psi'_{\mathbf{k}}, \tag{9}$$

with $\Phi_{\mathbf{k}}$, $\Psi_{\mathbf{k}}$ and $\delta_{\gamma\mathbf{k}}$ denoting amplitudes of corresponding perturbations in the form of plane waves, which are functions of the conformal time $\tau$ only, the CMB temperature anisotropies can be approximated by the formula

$$\begin{aligned} \frac{\delta T}{T_0}(\tau_0, \mathbf{l}) \;\approx\; & \int \frac{d^3k}{(2\pi)^{3/2}} \Bigg[ \left(\alpha(\mathbf{k}) + \beta(\mathbf{k})\frac{\partial}{\partial \tau_0}\right)_{\tau_{\rm rec}} e^{i\mathbf{k}\cdot\mathbf{l}(\tau_{\rm rec} - \tau_0)} + \\ & + \int_{\tau_{\rm rec}}^{\tau_0} \gamma(\tau, \mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{l}(\tau - \tau_0)} d\tau \Bigg] e^{-\Sigma k^2}, \end{aligned} \tag{10}$$

where the constant $\Sigma$ is defined as

$$\Sigma \equiv \frac{1}{6}\left(\frac{z(\tau_{\rm rec})}{Z}\right)^2 \frac{a(\tau_{\rm rec})}{a'(\tau_{\rm rec})}, \quad Z = \frac{\mathcal{E}_{2\rm S}}{k_{\rm B} T_0}, \tag{11}$$

where $z(\tau)$ is the redshift of radiation emitted at the conformal time $\tau$, $\mathcal{E}_{2\mathrm{S}} = 3.4eV$ is the ionization energy in the 2S state of the hydrogen atom and $k_{\mathrm{B}}T_0$ is the energy corresponding to the temperature of the CMB radiation today.

Since the CMB anisotropies are random, we must introduce correlation functions if we want to compare observations with the theory. The simplest quantity measuring statistical features of the CMB anisotropies is the two-point correlation function, which is conventionally encoded in the CMB angular power spectrum. Another important quantity is the three-point correlation function encoded in bispectrum, which measures the size and shape of non-Gaussianity.

The two-point correlation function given by the observed CMB temperature anisotropies is defined as

$$C(\theta) = \left\langle \frac{\delta T}{T_0}(\tau_0, \mathbf{l}_1) \frac{\delta T}{T_0}(\tau_0, \mathbf{l}_2) \right\rangle_{\mathbf{l}_1, \mathbf{l}_2}, \quad \mathbf{l}_1 \cdot \mathbf{l}_2 \stackrel{\mathrm{fix}}{=} \cos\theta, \tag{12}$$

where the brackets $\langle\ \rangle_{\mathbf{l}_1, \mathbf{l}_2}$ denote averaging over all directions $\mathbf{l}_1$ and $\mathbf{l}_2$ while keeping the angle between them equal to $\theta$. This is to be compared with the function $C(\theta)$ obtained by a different averaging

$$C(\theta) = \left\langle \frac{\delta T}{T_0}(\tau_0, \mathbf{l}_1) \frac{\delta T}{T_0}(\tau_0, \mathbf{l}_2) \right\rangle_{\mathrm{ens}}, \quad \mathbf{l}_1, \mathbf{l}_2\text{-fix: } \mathbf{l}_1 \cdot \mathbf{l}_2 \stackrel{!}{=} \cos\theta, \tag{13}$$

where the brackets $\langle\ \rangle_{\mathrm{ens}}$ denote averaging over the statistical ensemble of the CMB anisotropies or, equivalently, over all observers in the universe. The two-point correlation function given by (13) can be rewritten into the form

$$C(\theta) = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) C_l P_l(\cos\theta), \quad C_l = \left\langle |a_{lm}|^2 \right\rangle_{\mathrm{ens}}, \tag{14}$$

where $P_l$ are Legendre polynomials. Coefficients $C_l$ are called the *angular power spectrum* of the CMB.

The coefficients $\langle |a_{lm}|^2 \rangle_{\mathrm{ens}}$ are independent on $m$, because there is no preferred direction in the universe, but for the observer there is a statistical randomness of the CMB anisotropies, so the observed $\langle |a_{lm}|^2 \rangle$ are not equal for all $m$ for a given multipole moment $l$. The best estimation of these coefficients is then given by averaging

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2. \tag{15}$$

The corresponding error of the values of $C_l$ is then given by the expression known as the *cosmic variance*,

$$\frac{\Delta C_l}{C_l} = \sqrt{\frac{2}{2l+1}}. \tag{16}$$

This error is unavoidable, since for the observer there is available only one observation of the CMB radiation.

The angular power spectrum can be expressed in the form

$$C_l = \frac{2}{\pi} \int\limits_0^\infty \tau_l(k)^2 (2\pi)^3 \mathcal{P}_\Phi(k) \frac{dk}{k}, \tag{17}$$

where

$$\begin{aligned} \tau_l(k) \quad = \quad & \left[ \left. \frac{\alpha(\mathbf{k})}{\Phi^{(0)}_{\mathbf{k}}} \right|_{\tau_{\rm rec}} j_l(k(\tau_0 - \tau_{\rm rec})) + \left. \frac{\beta(\mathbf{k})}{\Phi^{(0)}_{\mathbf{k}}} \right|_{\tau_{\rm rec}} \frac{\partial}{\partial \tau_0} j_l(k(\tau_0 - \tau_{\rm rec})) + \right. \\ & \left. + \int\limits_{\tau_{\rm rec}}^{\tau_0} \frac{\gamma(\tau, \mathbf{k})}{\Phi^{(0)}_{\mathbf{k}}} j_l(k(\tau_0 - \tau)) d\tau \right] e^{-\Sigma k^2}, \end{aligned} \tag{18}$$

and the function $\mathcal{P}_\Phi$ called the primordial *power spectrum* defined by

$$\left\langle \Phi^{(0)}_{\mathbf{k}_1} \Phi^{(0)}_{\mathbf{k}_2} \right\rangle_{\rm ens} = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) k_1^{-3} \mathcal{P}_\Phi(k_1), \tag{19}$$

can be well approximated as a function proportional to a power of the comoving wavenumber,

$$\mathcal{P}_\Phi(k) \propto k^{n_s - 1}, \tag{20}$$

where the value of so-called *spectral index* $n_s$ is close to 1. When $n_s = 1$, the power spectrum is flat, and deviation of $n_s$ from 1 is called *spectral tilt.*

## Inflation

There are three basic problems which are not explained by the $\Lambda$CDM model, horizon problem, flatness problem and monopole problem. To solve these problems by inflation, the minimal number of e-foldings measuring the rate of expansion of the universe during inflation is 62 for inflation occurring at the energy scale of grand unified theory and 68 for the Planck energy scale.

A quantity which measures the deviation of evolution of the scale factor during inflation from the de Sitter solution is the slow-roll parameter,

$$\epsilon = -\frac{\dot{H}}{H^2}, \tag{21}$$

where $H = \dot{a}/a$. It is easy to see that if $\epsilon = 0$ we obtain exponential dependence of the scale factor on cosmological time. In terms of the conformal time this dependence reads

$$a = -\frac{1}{H\tau}, \tag{22}$$

where by convention $\tau \in (-\infty, 0)$. Since we have to avoid never ending inflation, we actually need the slow-roll parameter $\epsilon$ being a function which is small only for some limited period of time, during which the inflationary expansion occurs. In order to have sufficient number of $e$-foldings the inflation has to last long enough, so that the slow-roll parameter $\epsilon$ must be during inflation not only small but its time derivative must be small as well. For this reason we introduce the second slow-roll parameter $\eta$ as

$$\eta = \frac{\dot{\epsilon}}{\epsilon H}. \tag{23}$$

We may also consider the definitions of slow-roll parameters as equations for scale factor. Their solution for times such that $|\tau| \ll |\eta(\tau_{\rm ref})|^{-1}|\tau_{\rm ref}|$, where $\tau_{\rm ref}$ is some reference conformal time, can be approximated by

$$a(\tau) = a(\tau_{\rm ref})\left(\frac{\tau}{\tau_{\rm ref}}\right)^{-1-\epsilon(\tau_{\rm ref})}, \quad \epsilon(\tau) = \epsilon(\tau_{\rm ref})\left(\frac{\tau}{\tau_{\rm ref}}\right)^{-\eta(\tau_{\rm ref})}. \tag{24}$$

For the reference time it is conventional to choose the time when the longest mode of observational relevance today with the wavenumber $k_{\rm min} \sim a_0 H_0$ exits the horizon, i.e.,

$$\left|\frac{k_{\rm min}}{H(\tau_{\rm ref})a(\tau_{\rm ref})}\right| \sim |H_0\tau_{\rm ref}| = 1. \tag{25}$$

## Solid matter in cosmology

The universe is not filled with the perfect fluid. During the pre-recombination period interaction of photons with the baryonic matter caused viscosity which is effectively described by the additional non-diagonal terms in the stress-energy tensor. A more speculative example of an imperfect fluid in cosmology is a solid matter with nonzero shear modulus. The solid matter can be fully described with the use of the spatial *internal* or *body coordinates*, which by definition are always constant for a given element of the solid and kinematics of the solid is given by the mapping

$$f: \begin{array}{ccc} \text{space-time} & \to & \mathbb{R}^3 \\ x^\mu & \mapsto & \phi^I \end{array}. \tag{26}$$

The metric which measures angles and distances in the solid with respect to the internal coordinates then can be obtained by push-forward of the space-time metric tensor

$$B^{IJ} = -g^{\mu\nu}\phi^I{}_{,\mu}\phi^J{}_{,\nu}, \tag{27}$$

where the sign is chosen to obtain the positive metric, and it is called the *internal* or *body metric*. In order to apply the perturbation theory we consider the internal metric to have the form of a sum of background metric and perturbation,

$$B_{IJ} = B^{(0)}_{IJ} + \delta B_{IJ}, \tag{28}$$

where the unperturbed part $B^{(0)}_{IJ}$ corresponds to the solid in equilibrium.

The dynamics of the relativistic solid matter can be formulated with the use of the equation of state which determines the dependence of the energy density of the solid $\rho_{\mathrm{s}}$ on its deformation given by the perturbation part of the internal metric, $\rho_{\mathrm{s}} = \rho_{\mathrm{s}}(\delta B^{IJ})$. Considering a homogeneous and isotropic solid, the equation of state must be also homogeneous and isotropic, i.e. it has to be invariant under global internal rotations and translations. The equation of state can be rewritten as

$$\begin{aligned} \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{s}}^{(0)}} &= 1 + \frac{1}{2}\left(1 + w_{\mathrm{s}}\right)\delta B + \frac{1}{8}\left(\frac{\lambda}{\rho_{\mathrm{s}}^{(0)}} + 1 + w_{\mathrm{s}}\right)\delta B^2 + \\ &\quad + \frac{1}{4}\left(\frac{\mu}{\rho_{\mathrm{s}}^{(0)}} - 1 - w_{\mathrm{s}}\right)\delta B \cdot \delta B, \end{aligned} \tag{29}$$

where $\lambda$ and $\mu$ are *Lamé coefficients*, and $w_{\mathrm{s}}$ is the pressure to energy density ratio of the unperturbed solid matter.

## Radiation-like solid in pre-recombination era

Now we will study effects of presence of a solid matter with nonzero shear modulus filling the universe before recombination, reproducing [43]. We will restrict ourselves to the case, when the pressure to energy density ratio of this solid is $w_{\mathrm{s}} = 1/3$, the same as for radiation, and photons are coupled with this solid. We will call this solid matter the *radiation-like solid.* To study the dynamics of the relativistic solid matter we will use so-called *proper-time comoving gauge* [32], in which the (00)-component of the metric tensor is unperturbed and the shift vector of the radiation-like solid $\delta\mathbf{x}$ is zero as well as for all matter coupled with it. Therefore, in this gauge the scalar part of the perturbed FRWL metric takes the form

$$ds^{2\mathrm{S}} = a^2 \left\{ d\tau^2 + 2B_{,i} d\tau dx^i - \left[ (1-2\psi)\, \delta_{ij} - 2E_{,ij} \right] dx^i dx^j \right\}, \tag{30}$$

and the internal coordinates coincide with the space coordinates, $\phi^I(x) = \delta^I_i x^i$.

Equations for invariant metric perturbations $\mathcal{E}$ and $\mathcal{B}$, invariant perturbations describing the dark matter velocity and energy density $\hat{\vartheta}$ and $\hat{\delta}^{\mathrm{d}}$ and perturbation of nutrinos $\hat{\sigma}$ and $\hat{\delta}^{\nu}$ in the form of plane waves with wavenumber $k$ are

$$\begin{aligned}
\mathcal{E}' &= -\left(s^2 + 3\alpha_{\mathrm{bs}\gamma}\tilde{\mathcal{H}}^2\right)\tilde{\mathcal{B}} - \alpha_{\mathrm{bs}\gamma}\tilde{\mathcal{H}}\mathcal{E} - 3\alpha_{\mathrm{d}}\tilde{\mathcal{H}}^2\Theta - \alpha_{\mathrm{d}}\tilde{\mathcal{H}}\hat{\delta}^{\mathrm{d}} - && (31)\\
&\quad -3\alpha_{\nu}\tilde{\mathcal{H}}^2\Sigma - \frac{3}{4}\alpha_{\nu}\tilde{\mathcal{H}}\hat{\delta}^{\nu}, &&\\
\tilde{\mathcal{B}}' &= \left(3c_{s0}^2 + \alpha_{\mathrm{bs}\gamma} - 1\right)\tilde{\mathcal{H}}\tilde{\mathcal{B}} + c_{s||}^2\mathcal{E} + \Xi\mathcal{E}' + \alpha_{\mathrm{d}}\tilde{\mathcal{H}}\Theta + \alpha_{\nu}\tilde{\mathcal{H}}\Sigma, && (32)\\
\Theta' &= \alpha_{\mathrm{bs}\gamma}\tilde{\mathcal{H}}\tilde{\mathcal{B}} + (\alpha_{\mathrm{d}} - 1)\,\tilde{\mathcal{H}}\Theta + \alpha_{\nu}\tilde{\mathcal{H}}\Sigma, && (33)\\
\hat{\delta}^{\mathrm{d}\prime} &= \mathcal{E}' + s^2\left(\tilde{\mathcal{B}} - \Theta\right), && (34)\\
\Sigma' &= \alpha_{\mathrm{bs}\gamma}\tilde{\mathcal{H}}\tilde{\mathcal{B}} + \alpha_{\mathrm{d}}\tilde{\mathcal{H}}\Theta + \alpha_{\nu}\tilde{\mathcal{H}}\Sigma + \frac{1}{4}\hat{\delta}^{\nu}, && (35)\\
\hat{\delta}^{\nu\prime} &= \frac{4}{3}\mathcal{E}' + \frac{4}{3}s^2\left(\tilde{\mathcal{B}} - \Sigma\right), && (36)
\end{aligned}$$

where

$$\alpha_{\mathrm{bs}\gamma} = \frac{3}{2}\frac{\rho_+^{\mathrm{bs}\gamma}}{\rho}, \quad \alpha_{\mathrm{d}} = \frac{3}{2}\frac{\rho^{\mathrm{d}}}{\rho}, \quad \alpha_{\nu} = \frac{3}{2}\frac{\rho_+^{\nu}}{\rho}, \tag{37}$$

with the subscript $+$ standing for adding the pressure to the energy density. The prime in equations of motion denotes differentiation with respect to rescaled conformal time $\zeta = (\sqrt{2}-1)\tau/\tau_{\mathrm{eq}}$ with $\tau_{\mathrm{eq}}$ denoting the matter-radiation equality, $s = k\tau_*$, $\tau_* = \tau_{\mathrm{eq}}/(\sqrt{2}-1)$, $\tilde{\mathcal{H}} = a'/a$ with the redefined prime, $\tilde{\mathcal{B}} = \mathcal{B}/\tau_*$, $\Theta = \hat{\vartheta}/\tau_*$, $\Sigma = \hat{\sigma}/\tau_*$, $\Xi = 4\mathcal{N}/(3a\rho_+^{\mathrm{bs}\gamma}\tau_*)$, $\mathcal{N}$ is the viscosity coefficient of the system of baryon-radiation plasma and solid matter and $\rho^{\mathrm{bs}\gamma}$ is its energy density. These equations represent generalization of equations (4) in [29] valid in the long-wavelength limit only. The two sound speeds appearing in the system of equations, auxiliary sound speed $c_{s0}$ and longitudinal sound speed of the baryon-radiation plasma with radiation-like solid coupled to it $c_{s||}$, are defined as

$$c_{s0}^2 = \frac{K}{\rho_+^{\mathrm{bs}\gamma}}, \qquad c_{s||}^2 = c_{s0}^2 + \frac{4}{3}\frac{\mu}{\rho_+^{\mathrm{bs}\gamma}} = (1+3\xi)c_{s0}^2, \tag{38}$$

where $K$ is the compressional modulus and $\xi$ is the dimensionless shear modulus defined as

$$\xi = \frac{\mu}{\rho^{s\gamma}}, \tag{39}$$

$\rho^{s\gamma}$ being the sum of energy densities of solid and photons.

The CMB angular power spectrum is computed with the use of integral (17) and according to (18) we must know functions $\alpha(\mathbf{k})$, $\beta(\mathbf{k})$ and $\gamma(\tau, \mathbf{k})$ defined by (9). The numerical calculation of the CMB angular power spectrum simplifies considerably if we omit the integrated Sachs–Wolfe effect, but applicability of results reduces to small-scale sector, [34]. In such case, from three functions in (9) we need only first two of them evaluated at the time of recombination. They can be written in terms of proper-time comoving variables as

$$\begin{aligned} \alpha &= 2\tilde{\mathcal{H}}\left(\tilde{\mathcal{B}} + \frac{1}{s^2}\mathcal{E}'\right) + \left(\frac{1}{3} + \frac{6\xi}{s^2}\frac{\tilde{\mathcal{H}}^2}{1+X}\right)\mathcal{E}, \\ \beta &= -\frac{1}{\tau_*}\left[\frac{1}{s^2}\mathcal{E} + \frac{3}{s^2}\tilde{\mathcal{H}}\left(\tilde{\mathcal{B}} + \frac{1}{s^2}\mathcal{E}'\right)\right]', \end{aligned} \tag{40}$$

where $X = a/a_{\mathrm{eq}} = \zeta(\zeta + 2)$.

Define the variance of the coefficients of angular power spectrum due to the presence of radiation-like solid as

$$\frac{\Delta_\xi C_l}{C_l} = \frac{C_l(\xi) - C_l}{C_l}, \tag{41}$$

where $C_l$ and $C_l(\xi)$ are the coefficients of angular power spectrum in a universe without the radiation-like solid and with it respectively. The values of $\Delta_\xi C_l/C_l$ are compared with the cosmic variance (16) for the *Planck* values of cosmological parameters in Fig. 1.

The presence of radiation-like solid can be confirmed by observations only if $|\Delta_\xi C_l/C_l| > |\Delta C_l/C_l|$. As seen from Fig. 1, this holds for very high $l$, even if $|\xi|$ is small enough so that inverse inequality is valid for low multipole moments.

## Inflation driven by scalar field and solid matter

Now we reproduce [45], where the inflationary model including solid matter and single scalar field is studied. The model under consideration is generalization of so called *solid inflation* [12, 3] and the single-field model, with the matter Lagrangian density

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + F(X, Y, Z, \varphi), \tag{42}$$

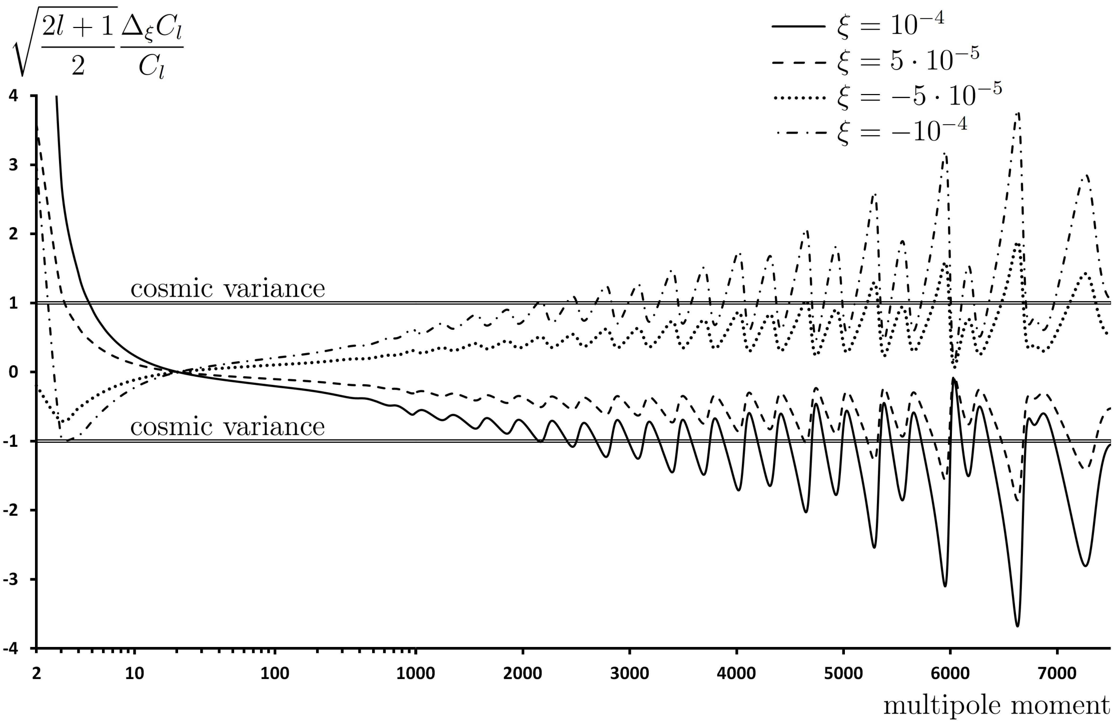


Fig. 1: The ratio of the variance of the coefficients of angular power spectrum due to the presence of radiation-like solid to the cosmic variance, depicted for given values of the shear modulus parameter $\xi$.

where $X$, $Y$ and $Z$ are defined by

$$X = B^{II}, \quad Y = \frac{B^{IJ}B^{IJ}}{X^2}, \quad Z = \frac{B^{IJ}B^{IK}B^{JK}}{X^3}, \tag{43}$$

and the internal coordinates $\phi^I$ appearing in (27) play the role of inflationary fields. This is the most straightforward combination of the two models and it can be considered as, for instance, a simple toy model of interactions of fields driving the solid inflation with fields of an effective field theory of the standard model.

In this inflationary model the slow-roll parameter is

$$\epsilon = p + q - \frac{1}{3}pq, \quad p = \frac{\dot{\varphi}^2}{2M_{\rm Pl}^2 H^2}, \quad q = X\frac{F_X}{F}, \tag{44}$$

where $p$ and $q$ are the slow-roll parameters of the single-field inflation and the solid inflation respectively. In this work we restrict ourselves to the special case in which both $p$ and $q$ are small.

From [31] we adopt definition of the scalar quantity $\zeta$ that parametrizes the curvature perturbations. The corresponding power spectrum $\mathcal{P}_\zeta(k)$ is defined by the two-point function in the late time limit, $\langle 0|\,\zeta_{k_1}\zeta_{k_2}\,|0\rangle = [\mathcal{P}_\zeta(k_1)/(2k_1^3)]\,(2\pi)^5\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2)$. The corresponding spectral tilt up to the leading order of the slow-roll approximation is

$$n_S - 1 = -2\frac{c_{L,e}^5\sigma p_e\epsilon_c^{(\delta\varphi)} + (\epsilon_e - p_e)\,p_c^{(\mathcal{U})}}{\epsilon_e + \left(c_{L,e}^5\sigma - 1\right)p_e}, \quad \sigma = e^{2N_{\min}\left(\epsilon_c^{(\delta\varphi)} - p_c^{(\mathcal{U})}\right)} \tag{45}$$

where $c_L = (1 + 2XF_{XX}/(3F_X) + 8(F_Y + F_Z)/(9XF_X))^{1/2}$ is the longitudinal sound speed, $p^{(\mathcal{U})} = p - c_L^2 Q + \frac{1}{2}\eta_Q + \frac{5}{2}\eta_L$, $\epsilon^{(\delta\varphi)} = \epsilon + 2p + \frac{1}{3}\frac{F_{\varphi\varphi}}{H^2}$, $Q = \epsilon - p$, and both $\eta_Q = \dot{Q}/(\epsilon H)$ and $\eta_L = \dot{c}_L/(c_L H)$ have been assumed to be of the first order in the slow-roll parameters. The subscript $c$ stands for quantities evaluated at the reference time satisfying the restriction (25), subscript $e$ stands for quantities evaluated at the time when the inflation ends, $\tau_e \approx 0^-$, $N_{\min}$ is the minimal number of e-foldings ($N_{\min} \sim 60$), and $(k_{\max}/k_{\min})^{2\left(p_c^{(\mathcal{U})} - \epsilon_c^{(\delta\varphi)}\right)}$, $k_{\max} \sim 3000k_{\min}$ being the maximal wavenumber corresponding to the highest observed multipole moment of the cosmic microwave background, and $c_{L,c}^{-2p_c^{(\mathcal{U})}}$ were replaced by one. (For example $3000^{0.01} \doteq 1.08$ and $0.1^{0.01} \doteq 0.98$.)

The tensor spectral tilt in our model is $n_T - 1 = 2c_{L,c}^2\epsilon_c - 2(1 + c_{L,c}^2)p_c$ and tensor-to-scalar ratio is $r = \mathcal{P}_\gamma/\mathcal{P}_\zeta = 4c_L^5\epsilon^2/\left[\epsilon + (c_L^5 - 1)\,p\right]$.

The three-point function of the scalar $\zeta$ can be computed with the use of the in-in formalism [42]. The scalar bispectrum $B_\zeta(k_1, k_2, k_3)$, defined by relation $\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B_\zeta(k_1, k_2, k_3)$, consists of two parts,

$$B_\zeta(k_1, k_2, k_3) = F_Y B_\zeta^Y(k_1, k_2, k_3) + \mathcal{N}_\zeta c_{L,c}^2 \tilde{F}\tilde{B}_\zeta(k_1, k_2, k_3), \tag{46}$$

parametrized by three independent parameters of the theory, $F_Y$, $\tilde{F} = X(F_{XY} + F_{XZ})$ and $\mathcal{N}_\zeta c_{L,c}$, where $\mathcal{N}_\zeta$ is a number of of the order of number of $e$-foldings. Following conventions of [50], we define $x = k_2/k_1$ and $y = k_3/k_1$ and describe the bispectrum by the function $x^2y^2B_\zeta(1, x, y)$ defined in region $1 - x \le y \le x$, $1/2 \le x \le 1$, $0 \le y \le 1$. Shapes of the functions $x^2y^2B_\zeta^Y(1, x, y)$ and $x^2y^2\tilde{B}_\zeta(1, x, y)$ are depicted in fig. 2. All functions in the figure are normalized to have value 1 in the equilateral limit, $x = y = 1$. Our model with the additional degree of freedom allows for a wider range of different shapes of the bispectrum than the solid inflation. The overall bispectrum peaks in the squeezed limit, unless $\tilde{F}/F_Y = (5/6)\mathcal{N}_\zeta^{-1}c_{L,c}^{-2}$, when it peaks in the equilateral limit instead.

Following the definition (4) in [51], we find the non-linearity parameter $f_{\rm NL}$ in the form

$$f_{\rm NL} = \epsilon_c\left(\frac{19415}{13122}\frac{1}{c_{L,c}^2}\frac{F_Y}{F} - \frac{5}{18}\mathcal{N}_\zeta\frac{\tilde{F}}{F}\right)/\left[\epsilon_c + \left(c_{L,c}^5 - 1\right)p_c\right]^2. \tag{47}$$

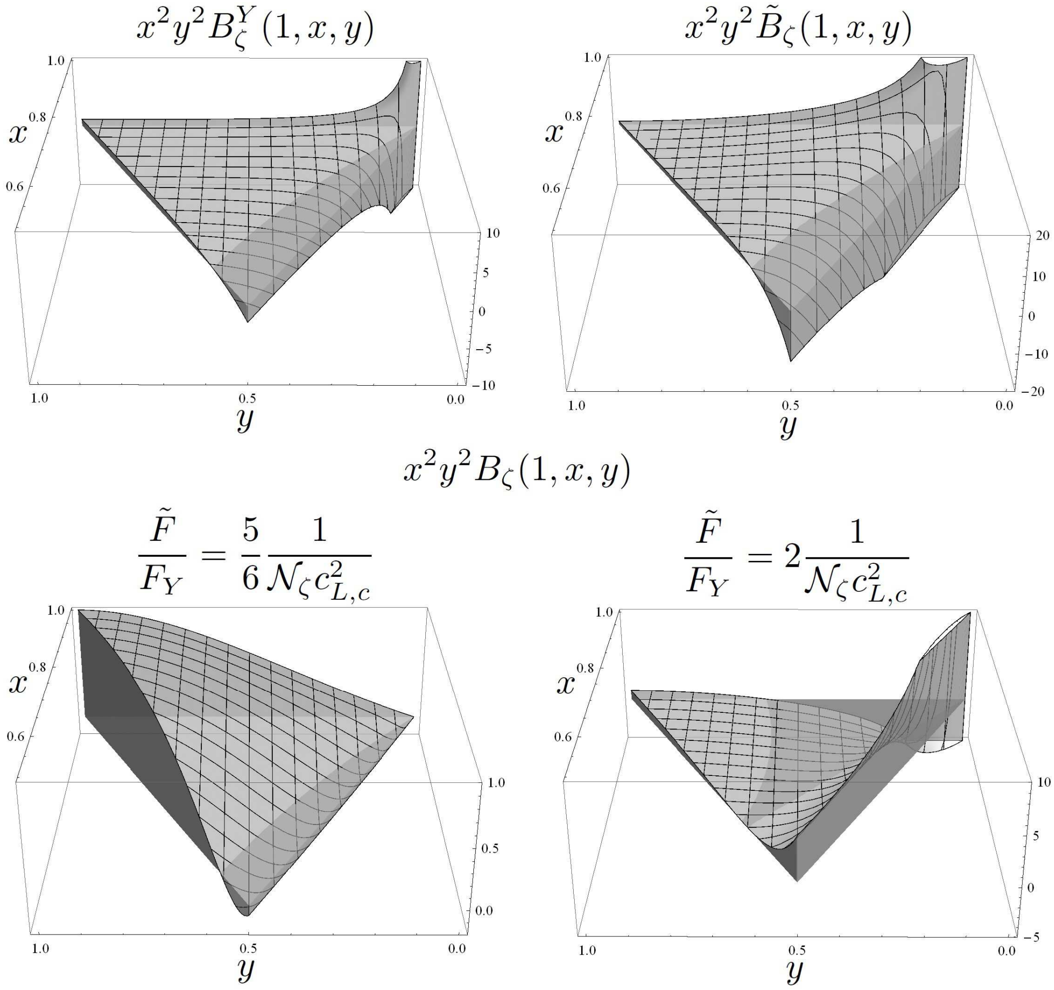


Fig. 2: Shapes of the scalar bispectrum. Flat triangles represent the zero plane.

We can see that if $\epsilon - p \sim \epsilon \sim p$, the non-linearity parameter is of the order $f_{\rm NL} \sim (F_Y/F)c_L^{-2}\epsilon^{-1}$, the same as for the solid inflation without the scalar field, or $f_{\rm NL} \sim \mathcal{N}_\zeta(\tilde{F}/F)\epsilon^{-1}$. Supposing that $c_L^5 \sim \epsilon$ we have $f_{\rm NL} \sim (F_Y/F)c_L^{-2}\epsilon^{-3}$ or $f_{\rm NL} \sim \mathcal{N}_\zeta(\tilde{F}/F)\epsilon^{-3}$ if $\epsilon - p$ is of the order $\epsilon^2$. The condition $\epsilon - p \lesssim \epsilon^2$ leading to an amplification of the non-linearity parameter can be rewritten as $q \ll p$, which means that the contribution of the solid matter to the overall stress-energy tensor is negligible in comparison to the contribution of the scalar field.

In our model the tensor bispectrum computed in the leading order of the slow-roll approximation does not differ from the tensor bispectrum in solid inflation. It is affected by presence of the scalar field only in the higher orders of the slow-roll approximation, which are not included in our work.

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