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Autoreferát dizertačnej práce

Advice Complexity of Online Problems

(Online algoritmy s pridanou informáciou)

na získanie vedecko-akademickej hodnosti philosophiæ doctor
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Introduction

In the classical approach to the design and analysis of algorithms it is assumed that an algorithm has the complete knowledge of the entire input. However, this assumption is unrealistic in a number of practical applications. In many fundamental real-world algorithmic problems the input comes sequentially and the output has to be produced continuously without knowing the entire input. The typical example of such problem is the paging problem, where the task is to maintain a two-level memory system consisting of a fast, but small, memory and a large, but slow, memory. The goal is to keep needed pages in the fast memory without knowing which pages will be requested in the future. Another example is scheduling of jobs on a set of machines, where the jobs arrive one by one and have to be scheduled immediately without any knowledge of future jobs.

The problems, for which the input becomes available in parts and the output needs to be produced before the next part of the input is known, are called *online problems*. The concept of online problem has been intensively investigated since its introduction in the late sixties [Gra66]. Despite the fact that online problems are studied for almost 50 years, the field still offers many fundamental unsolved problems. Moreover, a large number of online problems has been examined only for a very limited scope of parameters, thus providing only restricted understanding.

In the recent years, a research of usefulness of various kinds of information has been emerging. The main question in this type of research is to study information with respect to a particular problem, that is, how can additional information reduce the complexity of the problem. In an online setting, the research of additional information (about future request) is investigated using the notion of advice complexity introduced in [DKP09]. Advice complexity of a problem measures the amount of problem relevant information that is not available to the algorithm from the start of the computation. Many online problems has been studied with respect to their advice complexity recently, e.g., in [BKK⁺09, Kom11, EFKR11, BBF⁺14].

Goal

The thesis aims to continue in the existing research of advice complexity of online problems, and compare how some parameters of problems (such as allowing preemption, changing cost function) change the amount of relevant information that is hidden in the unknown input. In particular, two online problems are investigated: the disjoint path allocation problem, and the clique editing problem.

Results

For the disjoint path allocation problem, we investigate two versions, namely DPA_1, where every call has value 1, and DPA_ℓ, where the values of the calls are proportional to the lengths of the calls. For both of these versions we discuss preemptive and non-preemptive online algorithms with advice complexity measured either relative to the length of the input sequence n, or the length of the input path L. We provide several upper and lower bounds on advice complexity of optimal online algorithms. The achieved results are shown in Figure 1 in black, together with the known bounds shown in gray.

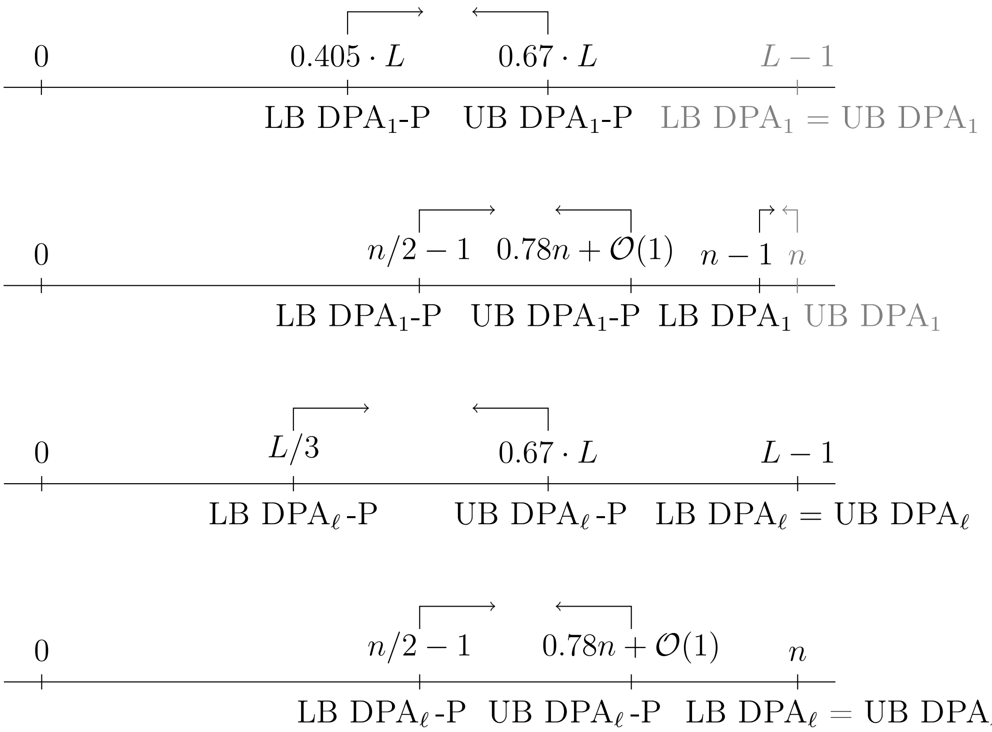

Figure 1: Lower and upper bounds for DPA_1 and DPA_ℓ with and without preemption.

Next, we provide some lower and upper bounds on the advice complexity of c-competitive algorithms. The bounds are valid for both preemptive and non-preemptive versions, DPA_1 and DPA_ℓ. The upper bound is an improvement over existing upper bound for the non-preemptive version of DPA_1. Previously known lower bound is better in the non-preemptive case, but it does not work in the preemptive setting.

Furthermore, we provide a general technique for construction of a $c(1+\varepsilon)$-competitive algorithm that uses a small number of advice bits from a c-competitive randomized algorithm, usable for all maximization online problems. The technique complements an analogous technique of [BKKK11] for minimization problems. Two examples of the usage of this technique are provided in the thesis – either for proving lower bounds for randomized online algorithms, or for showing upper bounds for online algorithms with advice.

The clique editing problem is investigated in both offline and online scenarios. For the offline version of the clique editing problem, we prove its NP-hardness even when restricted to bipartite graphs, and provide a 3.524-approximation algorithm. For the online clique editing problem, we prove that no algorithm can be competitive without advice, and provide asymptotically tight bounds on the advice complexity of optimal online algorithms.

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Abstrakt

Cieľom tejto práce je prispieť k súčasnému výskumu zložitosti rady online problémov. Zaoberáme sa dvoma problémami, problémom alokácie disjunktných ciest a problémom editovania na kliku. Prinášame dolné a horné hranice zložitosti rady optimálnych online algoritmov pre obidva tieto problémy. Pre problém alokácie disjunkných ciest je navyše zložitosť rady meraná vzhľadom na dĺžku vstupnej postupnosti dotazov alebo vzhľadom na dĺžku vstupnej cesty, a hovoríme o niekoľkých variantoch tohto problému (s alebo bez preemptovania, s konštantnými cenami dotazov alebo s cenami proporčnými dĺžke dotazu). Pre problém editovania na kliku skúmame aj ťažkosť offline verzie tohto problému.

V práci naviac poskytujeme techniku na konštruovanie dolných hraníc kompetitívnosti randomizovaných online algoritmov použitím dolných hraníc online algoritmov s radou pre maximalizačné problémy. Tiež analyzujeme zámenu kompetitívnosti a zložitosti rady pre problém alokácie disjunktných ciest.

Kľúčové slová: online problémy, zložitosť rady, preemptovanie, randomizácia, alokácia disjunktných ciest, editovanie na kliku, NP-ťažkosť, aproximácia

Abstract

This thesis aims to contribute to the recent research of advice complexity of online problems. Namely, we discuss two problems, the disjoint path allocation problem and the clique editing problem. We provide lower and upper bounds on advice complexity of optimal online algorithms for these problems. Moreover, in the case of the disjoint path allocation problem, the advice complexity is measured relative to either the length of the input sequence of requests, or the length of the input path, and we discuss several variants of the problem (with or without preemption, constant values of calls, or proportional to the length of a call). In the case of the clique editing problem, we discuss also the hardness of its offline version.

Furthermore, we provide a technique for constructing lower bounds on competitiveness of randomized online algorithms using lower bound of online algorithms with advice, for maximization problems, and some bounds on trade-off between competitiveness and advice complexity for disjoint path allocation problem.

Keywords: online problems, advice complexity, preemption, randomization, disjoint path allocation, call control, clique editing, NP-hardness, approximation