

**Univerzita Komenského v Bratislave**  
**Fakulta matematiky, fyziky a informatiky**

Veronika Gáliková

Autoreferát dizertačnej práce

**Kvantová mechanika v nekomutatívnom priestore**

na získanie akademického titulu philosophiae doctor

v odbore doktorandského štúdia:

**4.1.2 Všeobecná fyzika a matematická fyzika**

Bratislava, 4.5.2015

Dizertačná práca bola vypracovaná v dennej forme doktorandského štúdia na Katedre teoretickej fyziky a didaktiky fyziky FMFI UK

Predkladateľ:

Veronika Gáliková  
KTFDF FMFI UK  
Mlynská dolina  
842 48 Bratislava

Školiteľ:

prof. RNDr. Peter Prešnajder DrSc  
KTFDF FMFI UK  
Mlynská dolina  
842 48 Bratislava

Oponenti:

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Obhajoba dizertačnej práce sa koná ..... o ..... h pred komisiou pre obhajobu dizertačnej práce v odbore doktorandského štúdia vymenovanou predsedom odborovej komisie .....

4.1.2 Všeobecná fyzika a matematická fyzika

na .....

Predseda odborovej komisie:

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## Published papers, citations

**Coulomb problem in non-commutative quantum mechanics; Galikova, Veronika; Presnajder, Peter; Journal of Mathematical Physics Volume: 54 Issue: 5 Article Number: 052102**

Cited in:

Fradkin-Bacry-Ruegg-Souriau vector in kappa-deformed space-time Partha Guha, E. Harikumar, Zuhair N. S. Apr 8, 2015. 18 pp. e-Print: arXiv:1504.01897 [hep-th]

Dynamical Symmetry of the Zwanziger problem in Non-commutative Quantum Mechanics Juhi Rajhans. Dec 2, 2014. 9 pp. e-Print: arXiv:1412.1149 [math-ph]

Spectrum of the three dimensional fuzzy well N Chandra (IMSc, Chennai), H W Groenewald, J N Kriel (Stellenbosch U.), F G Scholtz (Stellenbosch U. & NITheP, Matieland), S Vaidya (Bangalore, Indian Inst. Sci.). Jul 22, 2014. 31 pp. Published in J.Phys. A47 (2014) 44, 445203 DOI: 10.1088/1751-8113/47/44/445203 e-Print: arXiv:1407.5857 [hep-th]

Noncommutative field theories on  $R^3$ ? : Toward UV/IR mixing freedom Patrizia Vitale (INFN, Naples & Naples U. & Orsay, LPT), Jean-Christophe Wallet (Orsay, LPT). Dec 2012. 31 pp. Published in JHEP 1304 (2013) 115, JHEP 1503 (2015) 115 ORSAY-PREPRINT-LPT-12-97 DOI: 10.1007/JHEP03(2015)115, 10.1007/JHEP04(2013)115 e-Print: arXiv:1212.5131 [hep-th]

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**Laplace-Runge-Lenz vector in quantum mechanics in noncommutative space; Galikova, Veronika; Kovacik, Samuel; Presnajder, Peter; Journal of Mathematical Physics Volume: 54 Issue: 12 Article**

**Number: 122106**

Cited in:

Fradkin-Bacry-Ruegg-Souriau vector in kappa-deformed space-time Partha Guha, E. Harikumar, Zuhair N. S. Apr 8, 2015. 18 pp. e-Print: arXiv:1504.01897 [hep-th]

Noncommutative Rd via closed star product V.G. Kupriyanov (ABC Federal U. & Tomsk State U.), P. Vitale (INFN, Naples & Naples U.). Feb 23, 2015. 27 pp. e-Print: arXiv:1502.06544 [hep-th]

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MICZ Kepler Systems in Noncommutative Space and Duality of Force Laws Partha Guha (Bose Natl. Ctr., Kolkata), E. Harikumar, Zuhair N. S. (Hyderabad U.). Apr 25, 2014. 19 pp. Published in Int.J.Mod.Phys. A29 (2014) 32, 1450187 DOI: 10.1142/S0217751X14501875 e-Print: arXiv:1404.6321 [hep-th]

On Laplace-Runge-Lenz Vector as Symmetry Breaking order parameter in Kepler Orbit and Goldstone Boson Manouchehr Amiri (Unlisted, IR). Apr 11, 2014. 17 pp. e-Print: arXiv:1405.2052 [physics.gen-ph]

Quantum gauge theories on noncommutative three-dimensional space Antoine Géré (Genoa U.), Patrizia Vitale (Naples U. & INFN, Naples), Jean-Christophe Wallet (Orsay, LPT). Dec 20, 2013. 20 pp. Published in Phys.Rev. D90 (2014) 4, 045019 DOI: 10.1103/PhysRevD.90.045019 e-Print: arXiv:1312.6145 [hep-th]

Reviewer Name: Simpao, Valentino Anthony; Mathematical Reviews/MathSciNet

Reviewer Number: 104391; Author: Galikova, Veronika; Kovacik, Samuel; Presnajder, Peter; Title: Laplace-Runge-Lenz vector in quantum mechanics in noncommutative space.

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The following paper has been submitted and published during the first year of my PhD studies. However, (although there of course is a connection to the problems related to this dissertation thesis), it covers more or less the issues that had been dealt with in my diploma thesis four years ago.

**Hydrogen atom in fuzzy spaces - Exact solution; Galikova, V.; Presnajder, P.; 7TH INTERNATIONAL CONFERENCE ON QUANTUM THEORY AND SYMMETRIES (QTS7); Book Series: Journal of Physics Conference Series Volume: 343 Article Number: 012096 Published: 2012**

Cited in:

Spectrum of the three dimensional fuzzy well N Chandra (IMSc, Chennai), H W Groenewald, J N Kriel (Stellenbosch U.), F G Scholtz (Stellenbosch U. & NITheP, Matieland), S Vaidya (Bangalore, Indian Inst. Sci.). Jul 22, 2014. 31 pp. Published in J.Phys. A47 (2014) 44, 445203 DOI: 10.1088/1751-8113/47/44/445203 e-Print: arXiv:1407.5857 [hep-th]

Noncommutative field theories on  $R^3$ ? : Toward UV/IR mixing freedom Patrizia Vitale (INFN, Naples & Naples U. & Orsay, LPT), Jean-Christophe Wallet (Orsay, LPT). Dec 2012. 31 pp. Published in JHEP 1304 (2013) 115, JHEP 1503 (2015) 115 ORSAY-PREPRINT-LPT-12-97 DOI: 10.1007/JHEP03(2015)115, 10.1007/JHEP04(2013)115 e-Print: arXiv:1212.5131 [hep-th]

Spectral triplets, statistical mechanics and emergent geometry in non-commutative quantum mechanics F.G. Scholtz (Stellenbosch U), B. Chakraborty (Stellenbosch U & Bose Natl. Ctr., Kolkata). Jun 2012. 15 pp. Published in

J.Phys. A46 (2013) 085204 DOI: 10.1088/1751-8113/46/8/085204 e-Print:  
arXiv:1206.5119 [hep-th]

Hydrogen atom in rotationally invariant noncommutative space By: Gnatenko,  
Kh. P.; Tkachuk, V. M. PHYSICS LETTERS A Volume: 378 Issue: 47  
Pages: 3509-3515

Eigenvalue problem for radial potentials in space with SU(2) fuzziness By:  
Mirahmadi, Marjan-S.; Fatollahi, Amir H. Journal of Mathematical Physics  
Volume: 55 Issue: 8 Article Number: 083518

Quantum mechanics with coordinate dependent noncommutativity By: Kupriy-  
anov, V. G. Journal of Mathematical Physics Volume: 54 Issue: 11 Article  
Number: 112105

## Summary

The geometry of a space that has "non-trivial structure on the small scales" expressed via some nontrivial commutation relations restricting simultaneous knowledge of relevant coordinates is said to be "non-commutative". There are many models suiting this description, distinguished by the form of the mentioned commutators of coordinates. The feature they have in common is that the notion of a single point has to be given up. The shorthand "NC" is used to denote anything related to such a space; for example we will write "NCQM" instead of "quantum mechanics in a space with non-commutative geometry".

This thesis provides an examination of how are prediction of standard quantum mechanics (QM) affected by introducing NC structure into the configuration space of the considered system (electron in the Coulomb potential in the present case).

The parameter controlling the extent of modification is denoted as  $\lambda$ . The coordinates in the NC space are realized via creation and annihilation operators acting in an auxiliary Fock space, this one being chosen in such a way that the rotational invariance of the system remains intact also in NCQM. The key relations that account for non-commutativity in the configuration space are

$$[x_i, x_j] = 2i \lambda \varepsilon_{ijk} x_k$$

Their importance for NCQM is analogous to that of the Heisenberg uncertainty relations in the QM case.

Analog of Schrödinger equation for hydrogen atom is found and analytically solved, both for bound states and scattering. Besides rotational invariance, the dynamical symmetry related to the conservation of NC analog of Laplace-Runge-Lenz vector is being used and the results obtained this way are in the full agreement with those given by "Schrödinger-like" approach.



**Achieved results:**

1. Complete formulation of the Coulomb-Kepler problem in NCQM with the potential  $U(r) \sim r^{-1}$
2. Exact solution of the problem and analysis of the results in regard to the dependence on the NC parameter  $\lambda$ .
3. Regular NC corrections to the Coulomb - Kepler problem in  $o(\lambda)$ , which lead to the standard spectrum in the correspondence limit  $\lambda \rightarrow 0$  (spectrum of the hydrogen atom , S-matrix)
4. New singular NC corrections  $o(\lambda^{-1})$ , which have no commutative analog. (Exact results)
5. Exact dynamical symmetry of the NC Coulomb-Kepler problem, which is an NC generalization of the Laplace-Runge-Lenz vector. Bound states found using algebraic method for both attractive and repulsive potential.
6. Algebraic derivation of the S-matrix related to the j-th partial wave, the predictions being in full agreement with those obtained as solutions of NC Schrödinger equation.

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