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Abstract model of the effectivity of human problem solving with respect to cognitive psychology and computational complexity

Autoreferát dizertačnej práce

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na získanie vedecko-akademickej hodnosti philosophiae doctor
v odbore doktorandského štúdia : 9.2.1 Informatika

Bratislava, 2015

Dizertačná práca bola vypracovaná v dennej forme doktorandského štúdia na Katedre informatiky Fakulty matematiky, fyziky a informatiky Univerzity Komenského v Bratislave.

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Obhajoba dizertačnej práce sa koná dňa .... o .... hod. pred komisiou pre obhajobu dizertačnej práce v odbore doktorandského štúdia vymenovanou predsedom odborovej komisie dňa .... v študijnom odbore

9.2.1 Informatika

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1 Motivation and goals of the thesis

Problem solving is probably the most common activity of all organisms, especially of humans. We deal with various problems throughout our lives, from infancy to adulthood. Therefore, it is not surprising that an extensive effort has been made to understand the cognitive processes responsible for this ability.

One possible research approach is to look at why some people are able to solve particular classes of problems while others are not, how they go about it, what makes some problems insolvable, and how is it possible that a small superficial change can make a very difficult problem into a trivial one (and vice versa). Through this approach psychologists and other scientists were able to shed a lot of light on many cognitive mechanisms of human problem solving.

However, there is difference between solving a problem by discovering a crucial relation that allows a quick solution and an extensive and long search through many possibilities. Undoubtedly, human beings have immense capacity for solving vast number of diverse and complex problems, but the pinnacle of our mental potential is not the capacity itself but rather the *effectivity* of this capacity. In this sense, we believe that there is still little research done on the effectivity of human problems solving, that is, we need to have a closer look at why some people are able to solve problems (much) *faster* than others, how and why are they able to discern and focus on the information crucial to the solution, and generally *what this effectivity depends on* and *how good it really is* when compared with some provable effective method.

In this thesis we attempt to give answers to the last two questions. And while these answers, if proved correct, provide new and important insights into human problem solving, they are of high value to AI research as well. Human brain is so far our only example of an effective general problem solving system (Langley, 2006), and by understanding the principles behind its effectivity, the development in AI (like cognitive architectures) can advance along more specific paths towards attaining human level abilities.

Here we formulate more particularly two our goals (mentioned above), and add some related ones:

1. Identify the cognitive processes (mechanisms, abilities) sufficient for successful human problem solving, and extract those of them that are the sources/roots of the effectivity of human problem solving

2. Compare the effectivity of human problem solving with an optimal strategy for problem solving (Solomonoff, 1986)

3. Using the previous results, propose an algorithm for problem solving

4. Analyse the optimal Solomonoff strategy when it is used mistakenly by imprecise information (note that it is used mistakenly quite often, since to gain enough precise information may be a hard problem)

The goals 1, 2, 3, and 4 have been achieved by the results from *Sections 3* & *4*, from *Section 5*, from *Section 6*, and from *Section 7*, respectively).

2 Introduction

Generally, the process of problem solving can be simplified to finding, testing, and applying the ideas (solution candidates). The effective problem solving can be then viewed as applying the *right* ideas at the *right* time. Such ideas can be consciously sought for, or they can come unconsciously like an automated procedure (tying shoelaces, for example). One way to find these ideas, albeit usually very ineffective one, is the aforementioned Blind search. On one hand, this exhaustive search is sometimes inevitable. For example, any problem with encrypted assignment is for a solver without the decryption key (or extensive cryptanalytic skills) practically insolvable without the exhaustive search (in fact, its insolvable either way as the space of decryption keys is usually huge, but that's not the point). Similarly, solving Rubik cube by trying all possible move sequences is highly inefficient as 20 moves are sufficient to solve any instance.

Therefore, Blind search is a universal but not an effective method, if there are too many solution candidates to check them one by one (which is usually the case). The other source of its ineffectivity is that the candidates are not tested in any problem related way (hence the name *Blind* search). Solomonoff (1986) in his problem solving system exploited a theorem in probability stating that if an appropriate order could be imposed on the solution candidates, then checking them in this order would yield *probabilistically optimal solution strategy.*

Theorem 2.1 (Solomonoff, 1986)**.** *Let $m_1, m_2, m_3, ...$ be candidates/ideas (or, in mechanical problem solving, strings) that can be used to solve a problem. Let p_k be probability that the candidate m_k will solve the problem, and t_k the time required to generate and test this candidate. Then, testing the candidates in the decreasing order $\frac{p_k}{t_k}$ gives the minimal expected time before a solution is found.*

Corollary 2.2. *The effectivity of problem solving depends on*

1. *Knowledge and experience,*
2. *Ability to generate – in the* ***effective order*** *(Theorem 2.1) and in a* ***short time*** *– the appropriate candidate ideas for solving the (sub)problems.*

3 Cognitive mechanisms related to the effective human problem solving

We propose a list of six cognitive abilities or mechanisms that, we argue, significantly help humans to generate in the effective order and in short time the appropriate candidate ideas for solving the (sub)problems. Thus, in our opinion these are the mechanisms that give rise to the effectivity of human problem solving, and as such should be implemented in cognitive architectures for this reason.

Proposition 3.1. *The following processes (mechanisms, abilities) are related to the effectivity of human problem solving (with respect to Theorem 2.1, especially to the second point of Corollary 2.2):*

1. ***Discovering similarities***
2. ***Discovering relations, connections and associations***
3. ***Generalization, specification***
4. ***Abstraction***
5. ***Intuition***
6. ***Context sensitivity and the ability to focus and forget***

4 General human problem solving mechanisms

In this section we analysed general human problem solving process and identified necessary and probably sufficient mechanisms for successful human problem solving.

Proposition 4.1. *The following processes (mechanisms, abilities) are necessary for successful human problem solving*

1. ***Discovering similar concepts*** *(representing information, experience, properties, problems, situations, models, ...)*
2. ***Discovering related, connected, and associated concepts*** *(representing information, experience, properties, problems, situations, models, ...)*
3. ***Manipulation***[1] ***with the information, problem, or situation or its representation*** *(e.g., transform, split, add or remove concepts, features, attributes, imagine, experiment, ...)*
4. ***Assessing***[1] ***the situation/progress and deciding*** *what to do next*
5. ***Incubation*** *(stop solving the problem)*

Furthermore, other cognitive processes not directly linked with problem solving, including

(a) *language processing,*
(b) *working memory processes (coordinating, monitoring, and executing the intended activities),*
(c) *ability to interpret/understand memories and experience,*

are (most likely) used as well.

Hypothesis 4.2. *The processes (mechanisms, abilities) from the Proposition 4.1 are sufficient for successful human problem solving.*

[1] By this we mean *application* of some method, procedure, heuristic, experience, common sense, logic, ... that performs manipulation/assessment action on something.

Additionally, we linked the effective cognitive mechanisms from *Proposition 3.1* with general problem solving mechanisms from *Proposition 4.1*, thus establishing first arguments about the effectivity of human problem solving.

Proposition 4.3. *In terms of human problem solving capabilities, the processes (mechanisms, abilities) from Proposition 3.1 together with (a)–(c) from Proposition 4.1 suffice to replace the processes (mechanisms, abilities) from Proposition 4.1.*

Hypothesis 4.4. *Tthe processes (mechanisms, abilities) from Proposition 3.1 together with the cognitive processes for*

- *(a) language processing,*
- *(b) working memory process (coordinating, monitoring, and executing the intended activities),*
- *(c) ability to interpret/understand memories and experience,*

are sufficient for successful human problem solving. Together, they are the mechanisms for general human problem solving.

5 The effectivity of human problem solving

In this section we put together our results from the *Sections 3* and *4* to analyse the scope and the roots of the effectivity of human problem solving.

Hypothesis 5.1. *In the probabilistic sense of Theorem 2.1, human problem solving is effective with respect to*

1. *solver's knowledge and experience,*
2. *quality of his processes, mechanisms, or abilities from Proposition 3.1.*

Given what we observe in the world, the human problem solving process is indeed fast (i.e., effective). Whence this effectivity comes from is still uncertain, but the results of our work summarized in *Hypothesis 5.1* suggests that the ***roots of the effectivity of human problem solving*** lie in

1. ***optimal problem solving strategy*** from Solomonoff (1986),
2. ***solver's knowledge and experience,***
3. ***quality of the solver's mechanisms from Proposition 3.1***,
4. ***language processing*** (in the sense of Baldo et al., 2005).

6 Human problem solving model

In this section we describe the effective general human problem solving process as an algorithm based on the general problem solving mechanisms from the *Section 4* and effective cognitive mechanisms from *Section 3*.

Proposition 6.1. *The human problem solving can be formulated as a process of the following steps.*

1. ***Represent the problem** and identify the difficulty*
2. ***Asses the problem model** for appropriateness and effectivity through macro process 4 from Proposition 4.1, or, if formulated as a separate problem (e.g., "How do I assess my problem model?"), through all mechanisms from Proposition 4.1*[2].
   (a) *If a sufficient model is available, go to step 3.*
   (b) *If there are more available sufficient models, select a subset and go to step 3.*
   (c) *Otherwise, formulate a new problem of finding better model (or of transforming the problem to a more promising problem*[3]*) , and go to step 1 (new problem to be solved).*
3. ***Find and asses idea(s)** to continue solving the problem (within the current model* [4]*)*
   (a) *If an applicable idea is available through the mechanisms from Proposition 4.1, go to step 4.*
   (b) *If more applicable ideas are available, select a subset and go to step 4.*
   (c) *Otherwise, formulate a new problem (of finding better idea, model, or of transforming the problem to a more promising problem), and go to step 1 (new problem to be solved).*
4. *Parallely **apply the (selected) idea**[5], if necessary interact with the world, and **update the problem model** accordingly.*
5. *If the problem is not solved, **return** to step 2 or 3, or (temporarily) give up.*

7 Mathematical aspects of the effective problem solving

In this section we consider the effect of interchanging two candidates with respect to the optimal Solomonoff strategy (*Theorem 2.1*) on the problem solving time and the number of candidates examined. We give several bounds on the error resulting from the mentioned interchange. However, since the values p_i and t_i from *Theorem 2.1* can be arbitrary, we examine three special restrictions (called expert, novice, and indifferent system, respectively) under which reasonable bounds can be achieved. Finally, we consider a modification of the Solomonoff strategy when the value of t_i for each i is not fixed. This modification models the case when we applied the same solution candidate (e.g., a method) to two or more similar problems each time solving the problem in different times.

[2]If new problem is formulated, the algorithm recursively iterates.

[3]That is, an easier, more known, simplified version of the problem, or its decomposition into sub-problems (e.g., independent – divide & conquer, dependent – dynamic programming)

[4]If there are more models selected, follow them by rotation.

[5]If there are more problem solving ideas selected, follow them by rotation.

Theorem 7.1 (Solomonoff, 1986). *If each bet costs 1 dollar, then betting in the order of decreasing value p_k (i.e., always taking the bet with highest win probability available) would give the greatest win probability per dollar.*

Remark 7.2. Note that the expected number of solution candidates examined is not given by E_S because we did not include the possibility that all of our solution candidates failed to solve the problem. The corrected value E_S is given by

$$E_S = \sum_{i=1}^{N} i \cdot \prod_{j=1}^{i-1} (1 - p_j) \cdot p_i + N \prod_{j=1}^{N} (1 - p_j).$$

This is because the probability of each candidate failing to solve the problem is $\prod_{j=1}^{N} (1 - p_j)$, while it takes us N trials to discover this.

Theorem 7.3 (Solomonoff, 1986). *In the general scenario, if one continues to select subsequent bets on the basis of maximum p_k/d_k, the expected money spent before winning will be minimal. Suppose we change dollars to some measure of time (t_k). Then, betting according to this strategy yields the minimum expected time to win.*

Remark 7.4. Note that the expected problem solving time is not given by E_T because, again, we did not include the possibility that all of our solution candidates failed to solve the problem. The corrected value E_T is given by

$$E_T = \sum_{i=1}^{N} \sum_{l=1}^{i} t_l \cdot \prod_{j=1}^{i-1} (1 - p_j) \cdot p_i + \sum_{l=1}^{N} t_l \cdot \prod_{j=1}^{N} (1 - p_j)$$

for the same reasons as in *Remark 7.2.*

Theorem 7.5. *Let $p_k - p_{k+1} = \theta > 0$ for some $k \in \{1, 2, ..., N-1\}$ (assuming $\{p_i\}_{i=1}^{N}$ to be ordered as before in the proof of the Theorem 7.1). Then, following the optimal Solomonoff strategy from Theorem 7.1 with $(k+1)^{th}$ solution candidate tried just before k^{th} (a solver's error) yields a sub-optimal expected number of solution candidates tried before either finding a solution or discovering that none of our solution candidates works, and the expected excess EXC can be quantified as follows*

$$EXC = \prod_{j=1}^{k-1} (1 - p_j) \cdot \theta.$$

Furthermore,

$$\theta \cdot e^{-S_{k-1}} \geq EXC \geq \theta \cdot (1 - S_{k-1} + (k-2) P_{k-1}^{\frac{k-1}{2k-4}}).$$

Theorem 7.6. *Exchanging the k^{th} and $(k+n)^{th}$ solution candidates in the optimal Solomonoff strategy from Theorem 7.1 (a solver's error) increases the expected number of solution candidates examined by at most the excess*

$$EXC = v_1 + v_2 + v_3$$

where

$$v_1 = k \cdot (p_{k+n} - p_k) \cdot Q_{k-1},$$

$$v_2 = \frac{p_k - p_{k+n}}{1 - p_k} \cdot \sum_{l=k+1}^{k+n-1} l \cdot Q_{l-1} \cdot p_l,$$

$$v_3 = (k+n) \cdot \frac{p_k - p_{k+n}}{1 - p_k} \cdot Q_{k+n-1}.$$

Corollary 7.7. *Let $p_k - p_{k+n} = \theta > 0$. Then, the term EXC from Theorem 7.6 can be upper bounded as follows*

$$EXC \leq \frac{\theta}{1 - p_k} \cdot (k+n) \cdot (np_{k+1} + 1)\, e^{-S_k}.$$

Theorem 7.8. *Let*

$$\frac{p_k}{t_k} - \frac{p_{k+1}}{t_{k+1}} = \theta > 0$$

for some $k \in \{1, 2, ..., N-1\}$ (assuming $\{\frac{p_i}{t_i}\}_{i=1}^{N}$ to be ordered as before in Theorem 7.3). Then following the optimal Solomonoff strategy from Theorem 7.3 with $(k+1)^{th}$ solution candidate tried just before k^{th} (a solver's error) yields a sub-optimal expected amount of time spent before either finding a solution or discovering that none of our solution candidates works, and the expected excess EXC can be quantified as follows

$$EXC = \prod_{j=1}^{k-1} (1 - p_j) \cdot t_k t_{k+1} \cdot \theta.$$

Furthermore,

$$\theta \cdot t_k t_{k+1} \cdot e^{-S_{k-1}} \geq EXC \geq \theta \cdot t_k t_{k+1} \cdot \left(1 - S_{k-1} + (k-2) P_{k-1}^{\frac{k-1}{2k-4}}\right).$$

Theorem 7.9. *Exchanging the k^{th} and $(k+n)^{th}$ solution candidates in the optimal Solomonoff strategy from Theorem 7.3 (a solver's error) increases the expected amount of time by at most the excess*

$$EXC = q_1 + q_2 + q_3$$

where

$$q_1 = T_{k-1} \cdot Q_{k-1} \cdot (p_{k+n} - p_k) + Q_{k-1} \cdot (t_{k+n} p_{k+n} - t_k p_k),$$

$$q_2 = \sum_{l=k+1}^{k+n-1} Q_{l-1} \cdot p_l \left(T_l \cdot \frac{p_k - p_{k+n}}{1 - p_k} + (t_{k+n} - t_k) \frac{1 - p_{k+n}}{1 - p_k}\right),$$

$$q_3 = T_{k+n} \cdot Q_{k+n-1} \cdot \frac{p_k - p_{k+n}}{1 - p_k}.$$

The first special case we would like to examine are domain experts (being a domain expert definitely helps problem solving). We can model the domain expert solver as a system of solution candidates where each solution candidate has *at least* some chance of solving a problem. That is,

$$\forall k: \ p_k \geq c, \quad \text{for some } c \in (0,1).$$

We are interested in upper bounds on the excess term EXC from *Theorems 7.6* and *7.9.*

Theorem 7.10. *The expected number of excessive solution candidates tried in the expert system before either finding a solution or discovering that none of our solution candidates works, which is expressed by the term EXC in Theorem 7.6, can be upper bounded as follows*

$$EXC \leq \theta \frac{p_{k+1}}{1-p_k} \frac{(1-c)^k}{c^2} \left(A - B(1-c)^{n-1}\right)$$

where $\theta = p_k - p_{k+n}$, *and*

$$A = 1 + kc \left[1 - \frac{1-p_k}{1-c} \frac{c}{p_{k+1}} \left(\frac{1-p_1}{1-c}\right)^{k-1}\right],$$

$$B = (1-c) + c(n+k)\left(1 - \frac{c}{p_{k+1}}\right).$$

Theorem 7.11. *Let T be the constant specified above. Then, expected increase of time in the expert system before either finding a solution or discovering that none of our solution candidates works, which is expressed by the term EXC in Theorem 7.9, can be upper bounded as follows.*

If $p_{k+n} - p_k \leq 0$, ***then***

$$EXC \leq T \cdot p_{max2} \cdot \frac{p_k - p_{k+n}}{1-p_k} \frac{(1-c)^k}{c^2} \left(A - B(1-c)^{n-1}\right)$$

where

$$A = 1 + kc \left[1 - \frac{1-p_k}{1-c} \frac{c}{p_{max2}} \left(\frac{1-p_{max1}}{1-c}\right)^{k-1}\right],$$

$$B = (1-c) + c(n+k)\left(1 - \frac{c}{p_{max2}}\right),$$

$$p_{max1} = \max\{p_1, ..., p_{k-1}\},$$

$$p_{max2} = \max\{p_{k+1}, ..., p_{k+n-1}\}.$$

If $p_{k+n} - p_k \geq 0$, ***then***

$$EXC \leq T \cdot \frac{p_{k+n} - p_k}{1-p_k} (1-c)^k \left(A - B(1-p_{max})^{n-1}\right)$$

where

$$A = k\frac{1-p_k}{1-c} - (1+kp_{max})\left(\frac{1-p_{max}}{1-c}\right)^k \frac{c}{p_{max}^2},$$

$$B = \frac{(1-p_{max})^{k+n-1}}{(1-c)^k}\left(k+n-\frac{c}{p_{max}^2}(1+p_{max}(k+n-1))\right),$$

$$p_{max} = \max\{p_1, ..., p_{k+n-1}\}.$$

Similarly, we can model the domain novice solver as a system where each solution candidate has *at most* some chance succeeding. That is,

$$\forall k: \; p_k \leq d, \quad \text{for some } d \in (0,1).$$

In this case we are interested in lower bounds on the excess term EXC from *Theorems 7.6* and *7.9*.

Theorem 7.12. *The expected number of excessive solution candidates tried in the novice system before either finding a solution or discovering that none of our solution candidates works, which is expressed by the term EXC from the Theorem 7.6, can be lower bounded as follows*

$$EXC \geq \theta \cdot \frac{p_{k+n-1}}{1-p_k}\frac{(1-d)^k}{d^2}\left(A - B(1-d)^{n-1}\right)$$

where $\theta = p_k - p_{k+n}$, *and*

$$A = 1 + kd\left[1 - \frac{1-p_k}{1-d}\frac{d}{p_{k+n-1}}\left(\frac{1-p_{k-1}}{1-d}\right)^{k-1}\right],$$

$$B = (1-d) + d(n+k)\left(1 - \frac{d}{p_{k+n-1}}\right).$$

Theorem 7.13. *Let T be the constant mentioned above. The expected increase of time before solving the problem in the novice system, which is expressed by the term EXC from the Theorem 7.9, can be lower bounded as follows.*

If $p_{k+n} - p_k \leq 0$, ***then***

$$EXC \geq T \cdot p_{min2} \cdot \frac{p_k - p_{k+n}}{1-p_k}\frac{(1-d)^k}{d^2}\left(A - B(1-d)^{n-1}\right)$$

where

$$A = 1 + kd\left[1 - \frac{1-p_k}{1-d}\frac{d}{p_{min2}}\left(\frac{1-p_{min1}}{1-d}\right)^{k-1}\right],$$

$$B = (1-d) + d(n+k)\left(1 - \frac{d}{p_{min2}}\right),$$

$$p_{min1} = \min\{p_1, ..., p_{k-1}\},$$
$$p_{min2} = \min\{p_{k+1}, ..., p_{k+n-1}\}.$$

If $p_{k+n} - p_k \geq 0$***, then***

$$EXC \geq T\frac{p_{k+n} - p_k}{1 - p_k}(1 - d)^k \left(A - B(1 - p_{min})^{n-1}\right)$$

where

$$A = k\frac{1 - p_k}{1 - d} - (1 + kp_{min}) \cdot \left(\frac{1 - p_{min}}{1 - d}\right)^k \frac{d}{p_{min}^2},$$

$$B = \frac{(1 - p_{min})^{k+n-1}}{(1 - d)^k} \left(k + n - \frac{d}{p_{min}^2}(1 + p_{min}(k + n - 1))\right),$$

$$p_{min} = \min\{p_1, ..., p_{k+n-1}\}.$$

We can also consider the case where the probabilities p_i are all approximately the same (denote this value p), and the values t_i are arbitrary. This case describes the situation where the solver has many similarly successful candidates (e.g., lots of very general methods of uncertain success), and he is required to choose. What effect on the expected problem solving time has the exchanging two candidates in this case?

Theorem 7.14. *Let p be the value mentioned above. The expected increase of time before solving the problem in the indifferent system, which is expressed by the term EXC from the Theorem 7.9, can be approximated as follows*

$$EXC \approx (t_{k+n} - t_k)(1 - p)^{k-1} \left(1 + (1 - p) - (1 - p)^{n+1}\right).$$

In real life problem solving a particular solution candidate (e.g., a method) could have been used to solve multiple similar problems each time consuming *a different amount of time.* Therefore, when the solver is considering a potential solution candidate, it has one cumulative probability of success (e.g., based on the experience and the strength of similarity/relatedness with the current problem model), but it can have multiple application times because of this possible application to the similar problems in the past. What order of examination of the solution candidates in this setting leads to the minimal expected time to find a solution? What if the solver remembers only an approximate average time?

Theorem 7.15. *Let s_k be a solution candidate which we in the past applied n_k times, and let $t_{k,j}$ be the execution time of the j^{th} application. Denote the mean execution time of the solution candidate s_k with Et_k:*

$$Et_k = \frac{t_{k,1} + ... + t_{k,n_k}}{n_k}.$$

If one continues to select subsequent candidates on the basis of maximum p_k/Et_k, then the expected time before solving the problem will be minimal (provided the problem can be solved by one of our candidates).

Abstract

The ability to solve problems effectively is one of the hallmarks of human cognition. Yet, in our opinion it gets far less research focus than it rightly deserves. In this paper we outline a framework in which this effectivity can be studied; we identify the possible roots and scope of this effectivity and the cognitive processes directly involved. More particularly, we have observed that people can use cognitive mechanisms to drive problem solving by the same manner on which an optimal problem solving strategy suggested by Solomonoff (1986) is based. Furthermore, we provide evidence for cognitive substrate hypothesis (Cassimatis, 2006) which states that human level AI in all domains can be achieved by a relatively small set of cognitive mechanisms. The results presented in this paper can serve both cognitive psychology in better understanding of human problem solving processes, and artificial intelligence in designing more human like intelligent agents.

Keywords: human problem solving, effectivity, mechanisms, artificial intelligence, cognitive architecture

Abstrakt

Jedna z najdôležitejších vlastností ľudského myslenia je určite schopnosť riešiť problémy efektívne. V tejto práci popisujeme súvislosti, v ktorých sa dá táto efektivity skúmať. Identifikujeme potenciálne korene a rozsah tejto efektivity a tiež kognitivne procesy, ktoré sú za ňu zodpovedné. Presnejšie, zistili sme, že ľudia vedia používať isté kognitívne mechanizmy spôsobom veľmi podobným optimálnej pravdepodobnostnej strategií na riešenie problémov, ktorú vo svojej práci použil Solomonoff (1986). Okrem toho, v práci ponúkame argumenty pre "Cognitive substrate hypothesis"(Cassimatis, 2006), ktorá hovorí, že umelá inteligencia na úrovni človeka može byť dosiahnutá pomocou relatívne malého počtu kognitívnych mechanizmov. Výsledky prezentované v tejto práci možu slúžiť kognitívnej psychologii pri lepšom porozumení ľudského procesu riešenia problémov, ako aj umelej inteligencií pri navrhovaní vysoko inteligentných agentov.

Kľúčové slová: ľudské riešenie problémov, efektivita, mechanizmy, umelá inteligencia, kognitívne architektúry

References

G. Altshuller. The Art of Inventing (And Suddenly the Inventor Appeared). *Technical Innovation Center, Worcester, MA*, 1994.

G. Altshuller. The innovation algorithm: TRIZ, systematic innovation and technical creativity. *Technical Innovation Center, Worcester, MA*, 2000.

J. Anderson. Rules of Mind. *Hillsdale, NJ: Lawrence Erlbaum Associates Inc.*, 1993.

J. Anderson. ACT: A simple theory of complex cognition. *American Psychologist 51.4*, 1996.

J. Anderson, D. Bothell, M. Byrne, S. Douglass, C. Lebiere, and Y. Qin. An integrated Theory of the Mind. *Psychological review 111.4*, 2004.

J. R. Anderson. A spreading activation theory of memory. *Journal of verbal learning and verbal behavior*, 22(3):261–295, 1983.

J. Baldo, D. Dronkers, N.F.and Wilkins, C. Ludy, P. Raskin, and J. Kim. Is problem solving dependent on language? *Brain and Language 92*, 2005.

M. Bassok. Analogical Transfer in Problem Solving, In R.Sternberg and J.E.Davidson (Eds.), The Psychology of Problem Solving. *Cambridge University Press*, 2003.

I. Bejar, R. Chaffin, and S. Embretso. Cognitive and psychometric analysis of analogical problem solving. *New York: Springer-Verlag*, 1991.

J. Carbonell. Derivational analogy and its role in problem solving. *Paper presented at the Third National Conference on Artificial Intelligence, Washington, DC*, 1983.

N. Cassimatis. A cognitive substrate for achieving human-level intelligence. *AI magazine, Vol. 27, No. 2*, 2006.

N. Cassimatis, P. Bignoli, M. Bugajska, S. Dugas, U. Kurup, A. Murugesan, and P. Bell. An architecture for adaptive algorithmic hybrids. *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on, 40(3)*, 2007.

W. G. Chase and H. A. Simon. Perception in chess. *Cognitive psychology*, 4(1), 1973.

A. Collins and E. Loftus. A spreading activation theory of semantic processing. *Psychological Review 82*, 1975.

A. M. Collins and M. R. Quillian. Retrieval time from semantic memory. *Journal of verbal learning and verbal behavior*, 8(2):240–247, 1969.

S. A. Cook. The complexity of theorem-proving procedures. In *Proceedings of the third annual ACM symposium on Theory of computing*, pages 151–158. ACM, 1971.

W. Croft and D. A. Cruse. *Cognitive linguistics.* Cambridge University Press, 2004.

A. d. C. DaSilveira and W. B. Gomes. Experiential Perspective of Inner Speech in a Problem-solving Context. *Paidéia (Ribeir?o Preto) [online], vol. 22, n. 51*, 2012.

J. Davidson. The suddenness of insight. In R.J.Sternberg and J.E.Davidson(Eds.), The nature of insight. *New York: Cambridge University Press*, 1995.

J. Davidson. Insights about insightful problem solving. In R.J.Sternberg and J.E.Davidson (EDs.), Psychology of problem solving. *Cambridge University Press*, 2003.

J. Davidson and R. Sternberg. The role of insight in intellectual giftedness. *Gifted child quarterly 28*, 1986.

I. J. Deary, L. J. Whalley, H. Lemmon, J. Crawford, and J. M. Starr. The stability of individual differences in mental ability from childhood to old age: follow-up of the 1932 Scottish Mental Survey. *Intelligence*, 28(1), 2000.

J. Dorfman, V. Shames, and J. Kihlstrom. Intuition, incubation, and insight: Implicit cognition in problem solving. In Geoffrey Underwood (Ed.), Implicit Cognition. *Oxford: Oxford University Press*, 1996.

W. Duch, R. Oentaryo, and M. Pasquire. Cognitive architectures: Where do we go from here. *AGI. Vol. 171*, 2008.

K. Duncker. On problem solving. *Psychological monographs 58*, 1945.

A. Ericsson. The acquisition of expert performance as problem solving. In R. Sternberg and J. Davidson, editors, *The psychology of problem solving*, pages 31–83. Cambridge University Press. Cambridge, England, 2003.

E. Fink. Automatic Evaluation and Selection of Problem-Solving Methods: Theory and Experiments. *Journal of Experimental and Theoretical Artificial Intelligence 16.2*, 2004.

K. Forbus. How minds will be built. *Advances in Cognitive Systems 1*, 2012.

D. Gentner and A. Stevens. *Mental models.* Lawrence Erlbaum Associates, Mahwah, NJ, 1983.

D. Gentner, M. Rattermann, and K. Forbus. The roles of similarity in transfer: Separating retrievability from inferential soundness. *Cognitive Psychology 25*, 1993.

M. Gick and K. Holyoak. Analogical problem solving. *Cognitive Psychology 12*, 1980.

B. Hayes-Roth and F. Hayes-Roth. A cognitive model of planning*. *Cognitive science*, 3(4), 1979.

B. Hayes-Roth, S. Cammarata, S. E. Goldin, F. Hayes-Roth, S. Rosenschein, and P. W. Thorndyke. Human planning processes. Technical report, DTIC Document, 1980.

D. Hebb. The organization of behaviour: A neuropsychological theory. *New York: Wiley*, 1949.

D. Hofstadter. Fluid concepts and creative analogies: Computer models of the fundamental mechanisms of thought. *London: Allen Lane, The Penguin Press*, 1997.

E. Hudlicka. Beyond Cognition: Modelling Emotion in Cognitive Architectures. *ICCM*, 2004.

J. Hummel and K. Holyoak. Distributed representation of structure: A theory of analogical access and mapping. *Psychological review, 104*, 1997.

M. Hutter. A theory of universal artificial intelligence based on algorithmic complexity. *arXiv preprint cs/0004001*, 2000.

M. Hutter. The fastest and shortest algorithm for all well-defined problems. *International Journal of Foundations of Computer Science*, 13(03), 2002.

M. Hutter. Optimality of universal Bayesian sequence prediction for general loss and alphabet. *The Journal of Machine Learning Research*, 4, 2003.

M. Hutter. *Universal artificial intelligence: Sequential decisions based on algorithmic probability*. Springer Science & Business Media, 2005.

M. Hutter. One decade of universal artificial intelligence. In *Theoretical foundations of artificial general intelligence*, pages 67–88. Springer, 2012.

C. Kaplan and H. Simon. In search of insight. *Cognitive Psychology 22*, 1990.

S. Kaplan. An introduction to TRIZ: The Russian theory of inventive problem solving. *Southfield, MI: Ideation International Inc.*, 1996.

M. Keane. *Modelling problem solving in Gestalt insight problem*. Milton Keynes, UK: The Open University, 1989.

M. Klamkin and D. Newman. Extensions of the Weierstrass Product Inequalities. *Mathematics Magazine, Vol. 43, No. 3*, 1970.

A. Koestler. The act of creation. *New York: Macmillan*, 1964.

J. Kolodner. An introduction to case-based reasoning. *Artificial Intelligence Review 6, Springer*, 1992.

L. Kotovsky and D. Gentner. Comparison and categorization in the development of relational similarity. *Child Development, 67*, 1996.

U. Kurup, P. Bignoli, J. Scally, and N. Cassimatis. An architectural framework for complex cognition. *Cognitive Systems Research 12.3*, 2011.

P. C. Kyllonen. Is working memory capacity spearman's g. In I. Dennis and P. Tapsfield, editors, *Human abilities: Their nature and measurement*, pages 49 – 75. Mahwah NJ: Lawrence Erlbaum Associates, Inc., 1996.

J. Laird. Extending the Soar cognitive architecture. *Frontiers in Artificial Intelligence and Applications 171*, 2008.

P. Langley. An adaptive architecture for physical agent. *The 2005 IEEE/WIC/ACM International Conference on. IEEE*, 2005.

P. Langley. Intelligent behavior in humans and machine. *American Association for Artificial Intelligence*, 2006.

P. Langley and D. Choi. A unified cognitive architecture for physical agent. *Proceedings of the National Conference on Artificial Intelligence. Vol. 21. No. 2*, 2006.

P. Langley and S. Rogers. An Extended Theory of Human Problem Solving. *Proceedings of the twenty-seventh annual meeting of the cognitive science society*, 2005.

P. Langley and N. Trivedi. Elaborations on a Theory of Human Problem Solving. *Poster Collection: The Second Annual Conference on Advances in Cognitive Systems*, 2013.

P. Langley, J. Laird, and S. Rogers. Cognitive architectures: Research issues and challenge. *Cognitive Systems Research 10.2*, 2009.

L. Levin. Universal sequential search problems. *Problemy Peredachi Informatsii*, 9(3), 1973.

M. Li and P. M. Vitányi. *An introduction to Kolmogorov complexity and its applications.* Springer Science & Business Media, 2009.

N. Maier. Reasoning in humans II: The solution of a problem and its appearance in consciousness. *Journal of Comparative Psychology: Learning, Memory, and Cognition 12*, 1931.

D. Medin and B. Ross. The specific character of abstract thought: Categorization, problem solving, and induction (Vol. 5). *Hillsdale, NJ: Erlbaum*, 1989.

S. Nason and J. Laird. Soar-RL: Integrating reinforcement learning with Soar. *Cognitive Systems Research 6.1*, 2005.

A. Newell. The heuristic of George Polya and its relation to artificial intelligence. *Carnegie Mellon University, Computer Science Department. Paper 2413*, 1981.

A. Newell and H. Simon. GPS, A program that simulates human thought. In E. A. Feigenbaum and J. Feldman, Computers and thought. *New York, McGraw-Hill*, 1963.

A. Newell and H. Simon. Human problem solving. *Englewood Cliffs, NJ: Prentice-Hall*, 1972.

A. Newell and H. Simon. Computer science as empirical inquiry: Symbols and search. *Communications of the ACM 19.3*, 1976.

A. Newell, J. Shaw, and H. Simon. Elements of theory of human problem solving. *Psychological Review 65*, 1958.

S. Ohlsson. Information processing explanations of insight and related phenomena. In M.T.Keane and K.J.Gilhooly (Eds.), Advances in psychology of thinking. *Cambridge, MA: Harvard University Press*, 1992.

L. Peterson and M. Peterson. Short-term retention of individual verbal items. *Journal of Experimental Psychology 58*, 1959.

G. Polya. How to solve it. *Doubleday, Garden City, NY, second edition*, 1957.

J.-C. Pomerol. Artificial intelligence and human decision making. *European Journal of Operational Research*, 99(1), 1997.

M. Quillian. Semantic memory. In M.Minsky (Ed.), Semantic information processing. *Cambridge, MA:MIT Press*, 1968.

S. Robertson. Problem Solving: Problem similarity. *Psychology Press*, 2003.

J. Schmidhuber. Optimal ordered problem solver. *Machine Learning*, 54(3), 2004.

J. Schmidhuber. Gödel machines: Self-referential universal problem solvers making provably optimal self-improvements. In *In Artificial General Intelligence*. Citeseer, 2005.

C. Seifert, D. Meyer, N. Davidson, A. Patalano, and I. Yaniv. Demystification of cognitive insight: Opportunistic assimilation and the prepared-mind hypothesis. *MIT Press*, 1994.

H. Simon. *Models of though (vol.2).* Yale University Press, New Haven, CT, 1989.

S. Slade. Case-based reasoning: A research paradigm. *AI magazine 12(1)*, 1991.

R. Solomonoff. A preliminary report on a general theory of inductive inference (revision of Report V-131). *Contract AF*, 49(639):376, 1960.

R. Solomonoff. A formal theory of inductive inference. Part I. *Information and control*, 7(1), 1964a.

R. Solomonoff. A formal theory of inductive inference. Part II. *Information and control*, 7(2), 1964b.

R. Solomonoff. Complexity-based induction systems: comparisons and convergence theorems. *Information Theory, IEEE Transactions on*, 24(4), 1978.

R. Solomonoff. Optimum sequential search. *Memorandum, Oxbridge Research, Cambridge, Mass*, 1984.

R. Solomonoff. The Application Of Algorithmic Probability to Problems in Artificial Intelligence. In L.N.Kanal and J.F.Lemmer (Eds.), Uncertainty in Artificial Intelligence. *Elsevier Science Publishers B.V. (North-Holland)*, 1986.

R. Solomonoff. Does algorithmic probability solve the problem of induction?, 2001.

R. Solomonoff. Algorithmic probability: Theory and applications. In *Information Theory and Statistical Learning*. Springer, 2009.

R. Solomonoff. Algorithmic probability, heuristic programming and AGI. In *Proc. 3rd Conf. on Artificial General Intelligence. Advances in Intelligent Systems Research*, volume 10, 2010.

R. Solomonoff. Algorithmic probability–Its discovery–Its properties and application to strong AI. *Randomness Through Computation: Some Answers, More Questions*, 2011.

V. Strassen. Gaussian elimination is not optimal. *Numerische Mathematik*, 13(4):354–356, 1969.

J. Veness, K. S. Ng, M. Hutter, and D. Silver. Reinforcement learning via AIXI approximation. *arXiv preprint arXiv:1007.2049*, 2010.

J. Veness, K. S. Ng, M. Hutter, W. Uther, and D. Silver. A monte-carlo aixi approximation. *Journal of Artificial Intelligence Research*, 40(1), 2011.

I. Wayne and P. Langley. Exploring moral reasoning in a cognitive architecture. *Proceedings of the Thirty-Third Annual Meeting of the Cognitive Science Society, Boston*, 2011.

R. Weisberg. Metacognition and insight during problem solving: Comment on Metcalfe. *Journal of experimental psychology: Learning, memory, and cognition, 18*, 1992.

R. E. Wing. Spatial components of mathematical problem solving (doctoral dissertation). 2005.

D. Woods. An Evidence-Based Strategy for Problem Solving. *Journal of Engineering Education 89*, 2000.

S. Wu. Some results on extending and sharpening the Weierstrass product inequalities. *Journal of Mathematical Analysis and Applications, Vol. 308, No. 2*, 2005.

Published work related to the thesis issues

F. Duris. Error bounds on the probabilistically optimal problem solving strategy. *Submitted to RAIRO - Theoretical Informatics and Applications*, 2015.