# Abstract

Algebraic methods are increasingly important in addressing problems in graph theory. Many problems, particularly those involving large or symmetric structures, are infeasible to solve using classical combinatorial approaches alone. This has led to a growing need to bridge informatics and algebra, where algebraic structures enable efficient computations and a deeper understanding of structural properties. This thesis examines problems related to optimal combinatorial structures (graphs, digraphs, and hypergraphs) using algebraic methods and computational tools.

For hypergraphs, we explore the hypergraphical regular representation problem, which generalizes the already solved problems of graphical and digraphical regular representations. Given a group, the aim is to construct a $k$-uniform Cayley hypergraph whose automorphism group equals the group's left regular action. We provide computational classification for groups of order up to 32 and extend our method for larger groups using dual hypergraph construction.

In the domain of digraphs, we analyze integer sequences derived from the orders of $k$-iterated line digraphs. We investigate several families of digraphs, where vertices correspond to words over an alphabet. This creates a natural bridge to the combinatorial problem of counting words while avoiding forbidden subwords. We compare the resulting sequences to the *OEIS* database and identify new ones.

Finally, we address questions related to the $(k, g)$-spectrum problem, determining the set of all possible orders of $k$-regular graphs with girth $g$. This generalizes the classical Cage Problem, which focuses on identifying the smallest such graphs. We present several construction methods (both algebraic and structural) to generate complete and incomplete spectra for various parameter pairs.

**Keywords:** graph theory, group actions, hypergraphical regular representation, integer sequences, $(k, g)$-spectrum