In this thesis, we investigate the qualitative and quantitative analysis of nonlinear parabolic equations arising from finance. First, we study the existence and uniqueness of solutions of nonlinear partial integro-differential equations (PIDEs) arising from the financial market. We consider Black–Scholes models for pricing options on underlying assets following a Lévy stochastic process with jumps. The existence and uniqueness results of the PIDE are presented in the scale of Bessel potential spaces using the theory of abstract semilinear parabolic equations in high-dimensional spaces. As an application in the one-dimensional space, we consider a general shift function arising from nonlinear option pricing models taking into account a large trader stock-trading strategy. We consider a PIDE, where the shift function may depend on a prescribed large investor stock-trading strategy function. Second, we analyze problems arising from stochastic dynamic optimization, which leads to a solution of a fully nonlinear evolutionary Hamilton-Jacobi-Bellman (HJB) equation. We consider the HJB equation arising from portfolio optimization selection, where the goal is to maximize the conditional expected value of the terminal utility of the portfolio. After a suitable transformation, the fully nonlinear HJB equation is transformed into a quasilinear parabolic equation whose diffusion function is obtained as the value function of a specific conic programming problem. We employ the monotone operator technique, Banach's fixed point theorem, and Fourier transform to obtain the existence and uniqueness of a solution to the general form of the transformed parabolic equation in a suitable Sobolev space in an abstract setting. We also presented some financial applications of the proposed result in one-dimensional space. Furthermore, the behavior of the solution corresponding to the nonlinear HJB equation is studied. We analyze the behavior of the solution with respect to two decision sets. Finally, we present numerical analyses of the parabolic equations using deep learning. Specifically, we employ the physics-informed deep operator network (PI-DeepONet) to approximate the solution operator of the parabolic equation associated with the HJB equation. Our qualitative analysis shows that PI-DeepONet can effectively learn the solution operator of the associated HJB equation.
***Keywords:*** **Hamilton-Jacobi-Bellman equation; Maximal monotone operator; Dynamic stochastic portfolio optimization; Lévy measure; Option pricing; Bessel potential spaces; Deep learning, PI-DeepONet**