

Abstract

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Abstract: This thesis is devoted to the analysis of infrared divergences in quantum field theory and to the mechanisms responsible for their cancellation in physically observable quantities. In the first part of the thesis, cutting rules are systematically derived using the largest time equation, first for scalar field theory and subsequently for fermions and massive vector bosons. The cancellation of infrared divergences is then studied explicitly in quantum electrodynamics for the process $e^+e^- \rightarrow \mu^+\mu^-$ at the next-to-leading order. One-loop corrections give rise to infrared singularities, which are shown to cancel only after the inclusion of real photon emission, in accordance with the Kinoshita-Lee-Nauenberg theorem. A central theoretical ingredient of the thesis is the formulation of holomorphic cutting rules. By expressing the squared transition amplitude in terms of contributions from a forward-scattering diagram with one or more on-shell cuts, this approach makes the analytic structure of singularities manifest. The distinction between normal and anomalous thresholds is emphasized and it is shown that anomalous thresholds play a crucial role in ensuring infrared finiteness.

Finally, the developed formalism is applied to axion production in the early Universe. The approach based on holomorphic cutting rules, together with the Kinoshita-Lee-Nauenberg theorem, allows one to identify the minimal set of contributions required to achieve infrared-finite results. Within this framework, axion production rates from scattering and decay processes are shown to remain finite in the limit of vanishing light-quark masses. The inclusion of anomalous threshold contributions leads to a qualitatively different picture from approaches based on thermal mass insertions, resulting in a balanced interplay between scattering and decay channels.

Key words: unitarity, holomorphic cutting rules, infrared divergences, the Kinoshita-Lee-Nauenberg theorem, the early Universe