

Abstract

FULOVÁ, Terézia: A conic optimization approach for solving matrix approximation problems [Dissertation Thesis], Comenius University in Bratislava, Faculty of Mathematics, Physics and Informatics, Department of Applied Mathematics and Statistics; Supervisor: doc. RNDr. Mária Trnovská, PhD., Bratislava, 2023, 150 p.

Matrix approximation problems are a subclass of constrained norm minimization problems. In this thesis, we handle a generalized formulation of matrix approximation problems to cover also well-known Procrustes problems. In general, matrix approximation problems have not been analyzed in a unified framework. The existing methods, which are designed to solve particular subclasses with a special structure of the feasibility set and a specific matrix norm in the objective. We aim to show that matrix approximation problems can be cast as conic programs with possible rank constraints. Therefore, we analyze various methods for solving rank-constrained optimization problems and propose a new solution algorithm based on using modified existing methods. Specifically, we address the problem of finding the nearest low-rank correlation matrix as well as different types of Procrustes problems, such as orthogonal, oblique, and semidefinite cases. We introduce a conic reformulation and demonstrate the correctness of this approach and the performance of our algorithm in numerous numerical experiments simulating real-life problems.

Keywords: conic optimization, matrix approximation problems, rank-constrained optimization problems, nearest correlation matrix, Procrustes problems