

UNIT 6

GEOMETRY

INTRODUCTION

Exercise 1. **Read the text, fill each gap with the correct preposition and find the synonyms of the following words.**

to assess:	_____	*evidence:*	_____
to comprise:	_____	*to describe:*	_____
realistic:	_____	*hypothesis:*	_____

ORIGINS OF GEOMETRY

Geometry began _____[1] a practical need to measure shapes. The word geometry means to "measure the Earth" and is the science _____[2] shape and size _____[3] things. It is believed that geometry first became important when an **Egyptian pharaoh** wanted to tax farmers who raised crops along the Nile River. To compute the correct amount _____[4] tax the pharaoh's agents had to be able to measure the amount of land being cultivated.

Around 2900 BC the first Egyptian pyramid was constructed. Knowledge of geometry was essential for building pyramids, which consisted _____[5] a square base and triangular faces. The earliest record _____[6] a formula for calculating the area of a triangle dates back to 2000 BC. **The Egyptians** (5000–500 BC) and **the Babylonians** (4000–500 BC) developed practical geometry to solve everyday problems, but there is no evidence that they logically deduced geometric facts _____[7] basic principles.

It was the **early Greeks** (600 BC–400 AD) that developed the principles _____[8] modern geometry beginning _____[9] **Thales of Miletus** (624–547 BC). Thales is credited _____[10] bringing the science of geometry from Egypt to Greece. Thales studied similar triangles and wrote the proof that corresponding sides of similar triangles are _____[11] proportion.

The next great Greek geometer was **Pythagoras** (569–475 BC). Pythagoras is regarded _____[12] the first pure mathematician to logically deduce geometric facts _____[13] basic principles. Pythagoras founded a brotherhood called the Pythagoreans, who pursued knowledge _____[14] mathematics, science, and philosophy.

Euclid of Alexandria (325–265 BC) was one _____[15] the greatest of all the Greek geometers and is considered _____[16] many to be the "father _____[17] modern geometry". Euclid is best

known _____[18] his 13-book treatise "*The Elements*", which is one of the most important works _____[19] history and had a profound impact _____[20] the development of Western civilization. Euclid outlined, derived, and summarized the geometric properties of objects that exist _____[21] a flat two-dimensional plane. This is why Euclidean geometry is also known_____[22] plane geometry. In plane geometry, the interior angles of triangles add up_____[23] 180^0, two parallel lines never cross, and the shortest distance between two points is always a straight line. The ancient Greek mathematicians developed the idea of an "axiomatic theory" which, for more than 2000 years, was regarded to be the ideal paradigm for all scientific theories.

The Muslim mathematicians made considerable contributions to geometry, trigonometry and mathematical astronomy and were responsible for the development of algebraic geometry.

There were no major developments _____[24] geometry until the appearance of **Rene Descartes** (1596–1650). Descartes combined algebra and geometry to create analytic geometry. Analytic geometry, also known as coordinate geometry, involves placing a geometric figure _____[25] a coordinate system to illustrate proofs and to obtain information using algebraic equations.

The next great development in geometry came _____[26] the development of non-Euclidean geometry, invented _____[27] **Carl Friedrich Gauss** (1777–1855) that generally refers _____[28] any geometry not based _____[29] the postulates of Euclid, including geometries _____[30] which the parallel postulate is not satisfied. The parallel postulate states that through a given point not on a line, there is one and only one line parallel _____[31] that line. Non-Euclidian geometry provides the mathematical foundation for Einstein's Theory of Relativity.

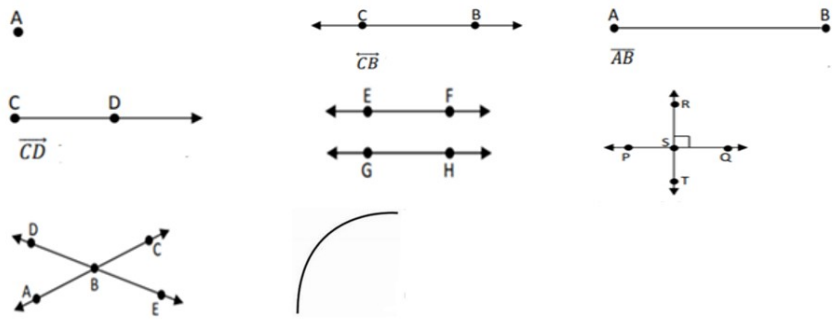
Exercise 2. **Match the following prefixes to their meanings. Give examples of derived words (used in mathematics/geometry) to each of the listed prefixes.**

~~Anti-~~	below	*against*	*anticlockwise*
Dis-	around	_____	_____
Semi-	middle	_____	_____
Inter-	before	_____	_____
Mid-	across	_____	_____
Peri-	remove	_____	_____
Sub-	~~against~~	_____	_____
Trans-	between	_____	_____
Pre-	half	_____	_____

PLANE, POINTS, LINES AND ANGLES

Exercise 3. Discussion.

a) What is a plane?
b) What is the minimum number of points required to define a plane?
c) Identify and explain the following symbols used in geometry: ∥,∠,⊥, °, ∟, ≅, ∼.
d) Identify the following plane geometry figures.



TERMINOLOGY

Exercise 4. **Match each verb from the left-hand column to its description in the right-hand column. Then use the verbs to complete the sentences in the following text putting them into the correct form. (Some of them are used more than once.)**

1.	to denote	*A.*	to be placed, to be situated
2.	to specify	*B.*	to indicate, to present, to demonstrate
3.	to extend	*C.*	to mention or speak about someone or something
4.	to show	*D.*	to say something in an exact and detailed way, to define
5.	to draw	*E.*	to find the size using standard units
6.	to refer to	*F.*	to continue over a particular distance
7.	to consist of	*G.*	to mean, to represent
8.	to measure	*H.*	to produce a picture of something, to sketch
9.	to lie	*I.*	to be formed of

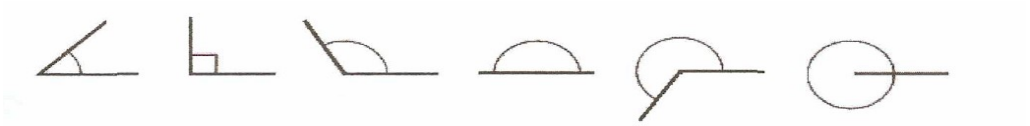
A **plane** in geometry is a flat surface that is infinitely large and has zero thickness. Two objects are coplanar if they both _____[1] in the same plane. We use points to _____[2] exact locations. **Points** are generally _____[3] by a number or letter. They are zero-dimensional; in other words, they have no length, width, or height. A **line** is simply an object in geometry that is characterized as a straight, thin, one-dimensional, zero-width object that _____[4] on both sides to infinity. A **straight line** is essentially just a line with no curves. It is _____[5] either using two points (labelled with capital letters) on a line with a double-headed arrow or using a single lower-case letter. Two points are always collinear since a straight line can always be _____[6] through two distinct points. Straight lines may be _____[7] in different directions and are given three names – *horizontal*, *vertical* and *oblique* or *slanting* lines. When we _____[8] to a line, we are talking about a straight line, unless otherwise noted.

A **line segment** _____[9] of two points (endpoints) and all the points on a straight line between them. A (geometric) **ray** has one endpoint and _____[10] from the endpoint to infinity in one direction on a line.

Exercise 5. **Discuss the following questions.**

1) How would you define (a) a midpoint of a line segment and (b) parallel lines?
2) How many points does a line have?
3) How many lines/line segments can be drawn through (a) a given point (b) two distinct points?

Exercise 6. **Properties of angles. Classify the angles below according to their magnitude and then complete the blanks in the text with appropriate expressions.**



Two rays having the same starting point form a geometric figure known as an _____[1]. The rays are called the **sides** of the angle and the common end point is called the **vertex** of the angle. A _____[2] **angle** is an angle whose sides are perpendicular. An _____[3] **angle** is less than 90°; an angle having more than 90° but less than 180° is an _____[4] **angle** and an angle having exactly 180° is called a _____[5] **angle**. A _____[6] **angle** is more than 180° but less than 360°. An angle where the final ray coincides with the initial ray completes a full rotation. Such an angle is called a _____[7] **angle** or a _____[8] **angle**.
We use capital letters or lower-case letters of the Greek alphabet to label angles. Any two lines that meet form two pairs of opposite angles called **vertical angels**, which always have the same degree measurement. When two parallel lines are intersected by a transversal, congruent pairs of angles are formed, such as **corresponding**, **alternate interior** and **alternate exterior** angles.

POLYGONS

Exercise 7. **Complete the text with the following phrases to get meaningful sentences. Then answer the questions below.**

finite number	*sides intersect*	*both the region*	*used classification*
line segments	*closed figure*	*different criteria*	*making up*

A polygon is a _____[1] made by joining _____[2], where each line segment intersects exactly two others. It is a closed planar figure composed of a _____[3] of sequential line segments. The straight line segments _____[4] the polygon are called its sides and the points where _____[5] are the polygon's vertices. The term polygon sometimes also describes the interior of the polygonal region or the union of _____[6] and its boundary. Polygons can be classified according to _____[7]. The most commonly _____[8] is based on the number of sides they have.

1) What is the difference between a convex and a concave polygon?
2) How would you define (a) an equilateral polygon, (b) an equiangular polygon, and (c) a regular polygon?

TRIANGLES

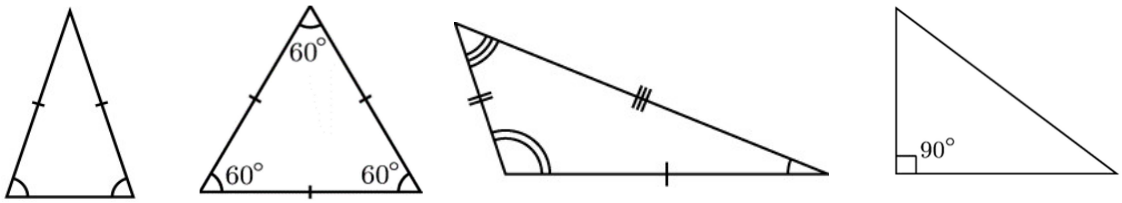
Exercise 8. **Put the words in brackets into the correct form and part of speech to complete the text.**

A **triangle** is one of the _____[1] (*base*) shapes in geometry. It is a polygon bounded by three straight lines (line segments), the sides, which _____[2] (*intersection*) at three points called the **vertices**. Any of the three sides may be _____[3] (*consideration*) the **base of the triangle**. The perpendicular _____[4] (*distant*) from the base to the opposite vertex is called an **altitude**. All three altitudes always intersect at the same point - the orthocentre of the triangle. A **median of a triangle** is a line segment _____[5] (*join*) a vertex to the midpoint of the opposite side. The **centroid of a triangle** is the point where the triangle's medians intersect.

Triangles can be classified according to the relative _____[6] (*long*) of their sides.

a) A **scalene triangle** is a triangle in which all the sides have _____[7] (*difference*) lengths and its internal angles are also _____[8] (*differ*).
b) In an **isosceles triangle**, two sides are of equal length. An isosceles triangle has also two internal angles _____[9] (*equality*).
c) In an **equilateral triangle**, all sides are of equal length. An equilateral triangle is also **equiangular**, i.e., all its internal angles are equal – _____[10] (*name*) 60°. It is also a regular polygon. In Euclidean geometry, the _____[11] (*summarise*) of the interior angles of a triangle is 180°.

Exercise 9. **Identify and describe the triangles below.**



Exercise 10. **Read the text and complete the Pythagorean Theorem.**

A right triangle (or right-angled triangle) has one of its interior angles measuring 90°. The side opposite the right angle is the ***hypotenuse***; it is the longest side of the right triangle. The other two sides are called the ***legs*** of the triangle. Right triangles obey the ***Pythagorean theorem***:

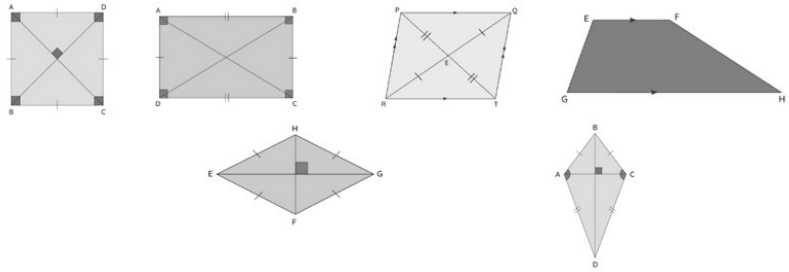
The _____[1] of the _____[2] of the lengths of the two legs is _____[3] to the square of the length of the _____[4]: $a^2 + b^2 = c^2$, where a and b are the _____[5] and _____[6] is the length of the hypotenuse.

QUADRILATERALS

Exercise 11. **Read the definition of a quadrilateral below. Match the various types of quadrilaterals with their description and complete the names of these quadrilaterals.**

The word 'Quadrilateral' is derived from a Latin word, in which, 'Quadra' means four and 'Latus' means sides. There are different types of quadrilaterals and they are uniquely identified on the basis of their distinct properties. Though, there are some properties that are common to all quadrilaterals:

- All quadrilaterals have four sides and four vertices.
- All quadrilaterals have two diagonals.
- The sum of the interior angles of quadrilaterals is 360°.



1) _____

a) two pairs of equal sides
b) two pairs of equal angles
c) the opposite sides are parallel
d) the diagonals bisect each other

2) _____

a) four sides of equal length
b) four right angles (90°)
c) the opposite sides are parallel
d) the diagonals bisect each other at right angles

3) _____

a) two pairs of equal sides
b) four right angles (90°)
c) the opposite sides are parallel
d) the diagonals bisect each other

4) _____

a) four sides of equal lengths
b) two pairs of equal angles
c) the opposite sides are parallel
d) the diagonals bisect each other at right angles

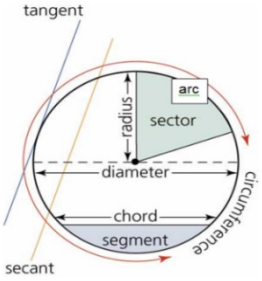
5) _____

a) one pair of parallel sides

6) _____

a) two pairs of equal sides
b) one pair of equal angles
c) the diagonals bisect at right angles

CIRCLE



A circle is a simple shape of Euclidean geometry consisting of those points in a plane which are equidistant from a given point called the centre.

Exercise 12. **Match the parts of a circle to their definition.**

_____	• the distance from the centre to any point on the circle. It is half the diameter.
_____	• the distance across the circle. The length of any chord passing through the centre. It is twice the radius.
_____	• the distance around the circle/the outer edge of a circle
_____	• a line segment linking any two points on a circle
_____	• a line passing a circle and touching it at just one point
_____	• a line that intersects a circle at two points
_____	• a part of the circumference
_____	• the area enclosed by an arc and two radii
_____	• the area inside a circle enclosed by an arc and a chord

AREA AND PERIMETER

Exercise 13. **Relationship between an area and a perimeter. Complete the text with any appropriate word/expression.**

_____[1] mathematics education literature reports _____[2] many learners and _____[3] adults adhere to the view that _____[4] with the same perimeter _____[5] have the same area. These studies interpreted learners' responses as resulting _____[6] a misunderstanding of the _____[7] *(a principle, an idea)* of area and perimeter, that is, figures with the same perimeter must have the same area and vice _____[8]. Furthermore, for a significant percentage of learners a predictable _____[9] *(the way in which two things are connected)* between area and perimeter is that _____[10] the area of a figure decreases or increases, the perimeter will _____[11] decrease or increase. Learners may not _____[12] that it is possible to have many rectangles _____[13] the same area, _____[14] different perimeters. _____[15] the perimeter is the same in a set of rectangles, then the area of those rectangles does not have to be the _____[16]. For example, a 3x4 and a 2x6 rectangles have the area of 12 _____[17] units, but their perimeters are 14 units and 16 units _____[18]. Rectangles with the same perimeter can have many _____[19] areas. For example, a 3x4 and a 2x5 rectangle _____[20] have a perimeter of 14 units, but their _____[21] are 12 square units and 10 square units respectively. (Abridged. France M. Machaba. (n.d.). The concepts of area and perimeter: Insights and misconceptions of Grade 10 learners. ERIC - Education Resources Information Center. Retrieved June 27, 2024, from https://files.eric.ed.gov/fulltext/EJ1209134.pdf)

GRAMMAR

CONDITIONALS

Conditionals are used to describe the result of a real or hypothetical condition. There are four main types of conditional in English. Each is used to indicate the likelihood of a situation occurring, or the likelihood that it would have occurred under particular circumstances.

Exercise 14. **Complete the rules (A – D) using expressions below and match them with suitable examples (1 – 4).**

second conditional	*imaginary consequences*	*zero conditional*	*first conditional*
may possibly happen	*third conditional*	*is unreal*	*general truths*

A The _____ is used to make statements about the real world, and often refers to _____, such as scientific facts.

B The _____, also known as the real conditional - describes a probable future situation. If the condition is fulfilled, the action in the main clause _____.

C The _____ refers to a situation that *is unlikely to happen* or _____.

D The _____ is used to talk about actions or events which did not happen in the past, and the _____.

1. If she had studied, she would have passed the exam.
2. If I were you, I would draw a different diagram; this one is not accurate.
3. A polygon is convex if no line that contains a side of the polygon contains a point in the interior of the polygon.
4. If you work hard, you will succeed.

Exercise 15. **Using the verbs in brackets, complete the sentences with their appropriate conditional forms (present, past or future).**

1) Two angles are called supplementary angles if the sum of their degree measurements _____ (*equal*) 180°.
2) If I _____ (*pass*) the test, I _____ (*get*) an ´A´ in the class. Instead, I got a ´C´. I really should have studied more.
3) If the circles _____ (*be*) separate, there _____ (*be*) four possible common tangents.
4) Even if we had changed only one of the coefficients on the right-hand side of the equation, the difference _____ (*become*) an infinitesimal of order 0,1 or 2.
5) If you submit your essay after the deadline, it _____ (not/*be*) marked.
6) If she _____ (*ask*) us, we _____ (*might know*) how to fix it, or we _____ (*could try*) at least. (She did not ask us.)

7) Completing an annual tax return _____ (*be*) a legal requirement if you _____ (*be*) self-employed.
8) If there had been doubts about the funding of the project, an investigation _____ (follow) immediately.
9) If I get 85% for the exam, I _____ (*have*) an A grade average overall.
10) If we changed the limits of integration, we _____ (*change*)the sign.

MIXED CONDITIONALS

Mixed conditionals are 'mixed' in the sense that the condition and the consequence are situated in different periods of time. In other words, the two parts of the sentence refer to different times. Mixed conditionals are commonly used by native speakers, and they always express unreal or impossible situations.

1. **Past Condition / Present Result:** to show an imaginary situation in the present based on a hypothetical condition in the past.

   If + past perfect (simple or continuous), ~ would (could, might) + infinitive.

*If I **had worked** harder at school (but I didn't, in the past), I **would have** a better job now.* (But I don't have now)

2. **Present Condition / Past Result:** - to show an imaginary situation in the past based on a hypothetical condition in the present.

   If+ past simple/continuous, ~ would (could, might) have + past participle

*If I **was** more diligent, I **would've finished** my degree at university.*

Exercise 16. **Using the verbs in brackets, complete the sentences with the appropriate conditional forms.**

1) If I _____ (*be*) a keen writer, I _____ (*become*) a journalist after leaving university.
2) If he _____ (*not/do*) well on the training course, he _____ (*be*) head of department now.
3) If Nick _____ (*turn in*) his application, he _____ (*work*) here now.
4) I _____ (*be*) able to translate the letter if my Italian _____ (*be*) better.
5) We _____ (*sack*) him months ago if we _____ (*not trust*) him.
6) If she _____ (*be*) born in the United States, she _____ (*not need*) a visa to work there.
7) If the students _____ (*have*) more time, they _____ (*finish*) their work yesterday.
8) If Mr. Smith _____ (*do*) an MBA, he _____ (*be*) on a higher salary now.
9) If I _____ (*learn*) German at school, I _____ (*be able*) to understand this journal article.
10) If we _____ (*be*) more realistic when planning projects, we _____ (*achieve*) all of the project's goals on time.

SOLID GEOMETRY



Exercise 17. **Read the text and fill the gaps with words of suggested meanings.**

At the time when Earth was discovered to be round rather than flat, spherical geometry began to _____[1] (*to begin to be known or noticed*) to aid navigators in mapping the land and water. However, even before Columbus, ancient Greek and Phoenician mariners used the ideas of spherical geometry in naval _____[2] (*the activity of searching and finding out about something*) of the world they knew. As much as 2000 years ago, the discovery of the Earth's curved _____[3] (*the outer or top part or layer*) was making a huge _____[4] (*the effect or influence that an event/situation has on someone or something*) on the way the Earth was viewed and mapped.

Observations of heavenly bodies were carried out in ancient Egypt and Babylon, mainly for astrological purposes and for making a calendar, which was important for organizing society. The Babylonians _____[5] (*to start something or cause it to happen*) the notion of dividing a circle into 360 degrees – speculation as to why 360 include that it was close to the number of days in a year, it was convenient to use in their hexadecimal system of counting. But, more important, the Babylonians developed a coordinate system for the _____[6] (*relating to the sky*) sphere.

The earliest known mathematical work that mentions spherical geometry is Euclid's *Phaenomena* (300 BC). The book uses _____[7] (*a formal statement that can be shown to be true by logic.*) from spherical geometry to solve astrological problems, such as *What is the length of daylight on a particular date at a particular latitude?* Euclid used thorough definitions and propositions from spherical geometry.

The definitions include: *A great circle is the intersection of the sphere by a plane through its centre* and *The intersection of the sphere by a plane not through the centre forms a (small) circle that is parallel to a unique great circle.* (Longitude lines and the equator on the Earth can serve as examples. Any pair of opposite points can be considered as the poles, and thus the equator and longitudes with respect to any pair of opposite points will be great circles.) The assumed _____[8] (*a statement that affirms or denies something and is either true or false)* include, for example: *Suppose two circles are parallel to the same great circle C but on opposite sides; then the two circles are equal if and only if they cut off from some other great circle equal arcs on either side C.*

It is natural for you to have some difficulty experiencing straightness on surfaces other than the 2-dimensional plane; it is likely that you will start to look at spheres and the curves on spheres as 3-dimensional objects. Imagining that you are a "2-dimensional bug" walking on a sphere helps you to _____[9] (*to get rid of*) your limiting extrinsic 3-dimensional vision of the curves on a sphere and to experience straightness intrinsically.
(Spherical geometry. (n.d.). Jim Wilson's. Retrieved June 27, 2024, from https://jwilson.coe.uga.edu/EMAT6680Fa11/Frailey/SphericalGeometryEssay_file)

TERMINOLOGY

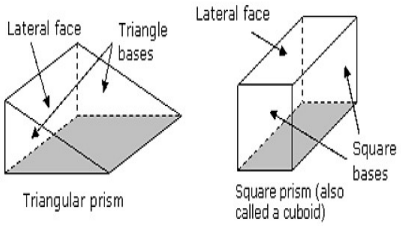
POLYHEDRA AND NON-POLYHEDRA

There are two main types of solids: **polyhedra**, e.g., prisms, pyramids and platonic solids and **non-polyhedra**, e.g., spheres, cylinders, tori (sing. torus), cones.

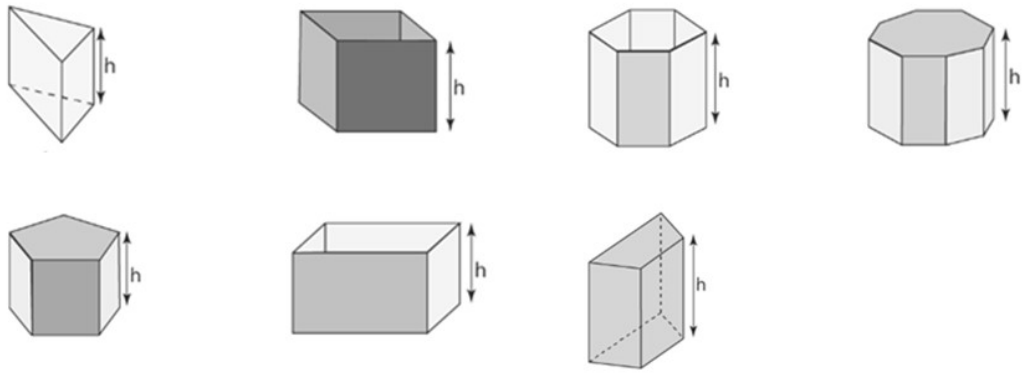
POLYHEDRA - PRISMS

A **polyhedron** (plural: polyhedra or polyhedrons) is a solid bounded by plane surfaces called faces (polygons). The faces intersect in straight lines called edges. Their vertices are vertices of the polyhedron. Lines connecting two vertices, not lying in one face are called diagonals of the polyhedron.
A convex polyhedron is one in which all diagonals lie within it.

Exercise 18. **Read the text and describe the prisms below.**



A **prism** is a polyhedron in which two of the faces (bases of the prism) are polygons that are congruent in parallel planes. If you cut a prism anywhere along its length, parallel to a base, its **cross-section** is the same – you would end up with two prisms. The side or lateral faces of a prism are parallelograms. A prism is triangular, pentagonal, hexagonal, etc. according as the base is a triangle, pentagon, hexagon, etc. A rectangular prism is called a **cuboid**.

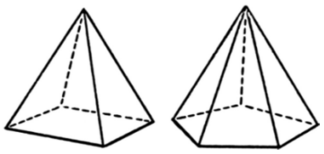


All the previous examples are ***regular prisms***, because the cross section is regular (in other words it is a shape with equal edge lengths, and equal angles.) Irregular prism has a base which is an irregular polygon with unequal side lengths. ***Irregular prisms*** have identical bases. However, the lateral faces are not identical.

POLYHEDRA - PYRAMID

Exercise 19. **While reading the text, fill in the gaps with the appropriate derived forms of following words:**

pentagon triangle basic rectangle square polygon perpendicular



A **pyramid** is a polyhedron formed by connecting a _____[1] base and a point, called the apex. Each base edge and apex form a triangle. When unspecified, the base is usually assumed to be _____ [2] and such polyhedron is called a square pyramid. A _____[3] pyramid has a rectangle for its base and its other faces are _____[4] regions. Similarly, a _____[5] pyramid has a pentagon as its base and triangular side faces. If the apex is _____[6] above the centre of the base, we have a right square pyramid, if the _____[7] is a regular polygon and the apex is above the centre of the polygon, we have a regular pyramid.

EULER'S FORMULA FOR POLYHEDRA

Exercise 20. **Complete the text with suitable verbs from the following list:**

express remember develop hold assert make be

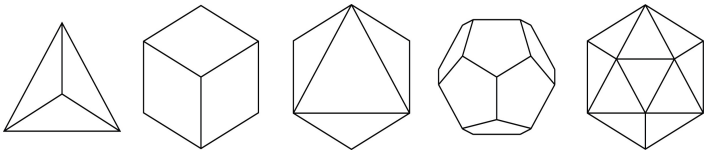
Leonhard Euler _____[1] an eighteenth-century Swiss mathematician and physicist, who _____[2] enormous contributions to several different fields. He is well _____[3] by his work in analytic geometry, trigonometry, geometry, calculus and number theory. Arguably, Euler's greatest achievement is the formula he _____[4], which _____[5] the relationship between the number of faces, vertices and edges in a solid. Euler's polyhedron formula _____[6] that for any convex polyhedron (it also _____[7] for some, but not all non-convex polyhedra), the number of faces (F) and vertices (V) added together is exactly two more than the number of edges (E). This can be written as **F + V - E = 2** or **F + V - 2 = E**.

POLYHEDRA - PLATONIC SOLIDS

Exercise 21. **Read the text and apply the Euler's formula to fill in the missing numbers in the table of Platonic solids.**

Platonic solids, also known as regular polyhedra, are five three-dimensional geometric shapes that have captured the imagination of mathematicians, artists and philosophers for centuries. Each solid is made up of identical faces, all of which are regular polygons, and the same number of faces meet at each vertex. There are exactly five such solids: - *the tetrahedron, cube, octahedron, dodecahedron and icosahedron.*

Plato, the ancient Greek philosopher, first described these five solids in his philosophical dialogues, associating each with one of the classical elements: earth, fire, air, water and the cosmos. For Plato, the solids symbolised the fundamental building blocks of the universe, representing the ideal forms from which all physical matter is derived.

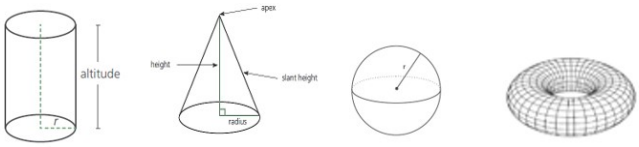


Platonic solid	Number of faces	Number of vertices	Number of edges	F + V – E = 2
tetrahedron				
hexahedron				
octahedron				
dodecahedron				
icosahedron				

THREE-DIMENSIONAL SHAPES WITH CURVES

Solid shapes which include a curved or round edge are not polyhedrons. Polyhedrons can only have straight sides. Many of the objects around you will include at least some curves. In geometry the most common curved solids are cylinders, cones, spheres and tori (sg. torus).

Exercise 22. **Read the text and complete the description of a cylinder, cone, sphere and torus with any appropriate expressions.**



(1) A **cylinder** is a geometrical figure in a 3-dimensional plane. It is made up of two congruent, _____[1] bases, that are usually _____[2], and a curved _____[3] surface that meets the bases at their perimeters. The centre line merging the circular _____[4] is the axis of the cylinder. The height is the _____[5] distance between the bases. As the bases are circles, the curved surface is like a _____[6]; hence the cylinder is a combined figure including circles and rectangles. Unlike some other 3D shapes, the number of _____[7] of a cylinder is zero. An object holding a shape like a cylinder is said to be cylindrical.

(2) A **cone** is a solid 3D geometric shape that is formed by the _____[8] of a right triangle around one of its right sides. The common point to which the set of line segments connect to at the top of the figure is the _____[9] or vertex of the cone, whilst the _____[10] at the bottom is the cone's base. The vertical side that remains fixed during the rotation of the right triangle is the axis (or _____[11], height occasionally) of the cone. Finally, the line segment connecting the vertex with any point on the circumference of the base's circle is called the _____[12].

(3) A **sphere** is a round object that has three dimensions. A sphere, unlike other three-dimensional shapes, does not contain any _____[13] or vertices. Each point on the sphere's _____[14] is located at the same distance from the sphere's centre. Consequently, the distance from the centre to the sphere's surface is the same in every place along its _____[15]. This particular distance is referred to as the sphere's "radius." A sphere must be perfectly_____[16] to meet the definition of a spherical. This implies that the distance between a sphere's exterior surface and centre is constant.

(4) A **torus** is a surface of revolution generated by revolving a _____[17] in three-dimensional space one full revolution about an axis that is coplanar with the circle.

Exercise 23. **Describe the following 3D figures. What is the difference between them?**



Exercise 24. **VOLUME: Complete the text with the following phrases.**

number of cubic *integral calculus* *one-dimensional* *more complicated*

Volume is the quantity of three-dimensional space enclosed by some closed boundary, for example, the space that a substance (solid, liquid, gas, or plasma) or shape occupies or contains. Volume is often quantified numerically using the SI derived unit, the cubic metre.
Volumes of some simple shapes, such as regular, straight edged, and circular shapes can be easily calculated using arithmetic formulas. The volumes of _____ shapes can be calculated by _____ if a formula exists for the shape's boundary.
_____figures (such as lines) and two-dimensional shapes (such as squares) are assigned zero volume in the three-dimensional space. Hence, when we find the volume of a space figure or a solid, we are finding the _____ units enclosed by the given space figure.

Exercise 25. **Form adjectives describing the solid figures below and complete the sentences:**

cylinder	_____	pyramid	_____
cube	_____	prism	_____
cone	_____	sphere	_____
ellipse	_____	hemisphere	_____

1) All the stones were of a _____ shape with sizes 10 by 25 by 50 cm.
2) All stars are _____.
3) Moving parts in the engine are called so because they are really _____.
4) The four possible _____sections are a circle, ellipse, parabola and hyperbola.
5) Comets orbit the Sun at extreme _____paths.

6) Pyrite is an example of a relatively rare _____crystal structure.
7) _____photography, also known as fisheye photography, is taken looking upward through an extreme wide-angle lens.
8) _____ancient buildings can be seen in Africa, Eurasia and America.

REVISION

Exercise 1. Fill the gaps with suitable expressions.

A. A _____[1] is the set of all points between and extending beyond two points. In most geometries, it does not have formal properties beyond length, its single dimension.
B. A _____[2] has one end point and may be extended infinitely in only one direction from that end point. A _____ _____[3] has two end points and it cannot be extended at all. We can _____[4] (*name)* a line using a lowercase letter such as *a, b,* or *c* or using two _____[5] (*not the same, different*) points on the line, for *example AB, AC, BC*, and so on.
C. A _____[6] is a line that deviates from straightness in a smooth, continuous fashion.
D. _____[7] lines are lines that are at right angles (90°) to each other, they intersect at right angles. _____[8] lines are lines on a plane that never meet. They are always the same distance apart.
E. Two rays having the same starting point form a geometric figure known as an _____[9]. The rays are called the _____[10] and the common end point is called the_____[11].
F. An angle measuring exactly 90 degrees is a _____[12] angle. Its sides are _____[13]. An angle measuring exactly 180 degrees is a _____[14] angle. An angle measuring between 90 and 180 degrees is referred to as an_____[15] angle. An angle measuring between zero and 90 degrees is an_____[16] angle.
G. A _____[17] is any closed planar figure composed of a finite number of sequential line segments. The straight line segments are called its _____[18] and the points where the sides intersect are its _____[19]. A polygon which has all sides mutually congruent and all angles mutually congruent is called a _____[20] polygon. A polygon that has all interior angles less than 180° is referred to as a _____[21] polygon. A polygon that has one or more interior angles greater than 180° is referred to as a_____[22] polygon. An_____[23] polygon has all interior angles equal. It is not necessarily also a regular polygon (rectangle has the same angles but different sides).
H. A _____[24] is one of the basic shapes in geometry. It is a polygon bounded by three straight lines (line segments), the sides, which intersect at three points called the_____[25] .The perpendicular distance from the base to the opposite vertex is called an _____[26]. In Euclidean geometry the sum of the angles of a triangle is _____[27].

   Triangles can be classified according to the relative lengths of their sides:
   A _____[28] triangle is a triangle in which all the sides have different lengths, and its internal angles are also different. In an_____[29] triangle two sides are of equal length. It has also two internal angles equal. In an_____[30] triangle all sides are

of equal length. It is also _____[31], i.e., all its internal angles are equal – namely 60°. It is also a _____[32] polygon.

A right triangle has one of its interior angles measuring _____[33]. The side opposite the right angle is the_____[34]; it is the longest side of the right triangle. The other two sides are called the _____[35] of the triangle.

A triangle that has all interior angles measuring less than 90° is an_____[36] triangle. A triangle that has one angle that measures more than 90° is an _____[37] triangle.

I. Two triangles are congruent if their corresponding _____[38] are equal in length and their corresponding _____[39] are equal in size.
J. Square and rhombus are parallelograms that have _____[40] (*neighbouring*) sides of equal length.
K. _____[41] of a parallelogram is equal to the product of its base and height (altitude): that is, A=*b.h.*
L. A _____[42] is a closed shape and a type of polygon that has four sides, four vertices and four angles. It is formed by joining four non-collinear points.
M. In a _____[43], only one pair of the opposite side is parallel to each other.
N. A six-sided polygon having all its sides and six angles equal is called a regular _____[44].
O. The_____[45] of a polygon is the sum of measures (sizes) of all its sides.
P. To _____[46] means to intersect exactly in the middle, dividing the line into two equal parts (halves).

Exercise 2. Read the following text on circle and fill in the missing terms.

A circle is a simple shape of Euclidean geometry consisting of those points in a plane which are equidistant from a given point called the _____[1].The common distance of the points of a circle from its centre is called its . _____[2].
Circles are simple closed curves which divide the plane into two regions, the interior and exterior. The_____[3] of a circle is the perimeter of the circle. The_____[4] of a circle is the length of a line segment whose endpoints lie on the circle, and which passes through the centre of the circle. A _____[5] of a circle is a line segment whose two endpoints lie on the circle. A _____[6] to a circle is a straight line that touches the circle at a single point. A _____[7] is an extended chord: a straight line cutting the circle at two points. An _____[8] of a circle is any connected part of the circle's circumference (boundary). A _____[9] is a region bounded by two radii and an arc lying between the radii, and a _____[10] is a region bounded by a chord and an arc lying between the chord's endpoints.

Exercise 3. Fill the gaps below with suitable expressions/phrases.

Solid geometry is the traditional name for geometry of three- _____[1] Euclidean space. It deals with properties of points, lines, and solids in space. There are two main types of solids: _____[2] **-** (they must have flat faces) and _____[3], e.g., spheres, cylinders, tori, cones.

- _____[4]: is a flat side of a 3-dimensional object.

- _____[5]: is one of two parallel, congruent sides of an object.
- _____[6]: is the intersection of two faces on a solid object. This is a line.

A **prism** is a polyhedron in which two of the faces (bases of the prism) are _____[7] that are congruent in parallel planes. A **pyramid** is a solid shape that has a polygonal base and sides that are _____[8] .The _____[9] (adj.) sides meet at a point called the _____[10]. The base of a polygon can be any polygon but most often is either a triangle or a quadrilateral.

The Platonic solid with four polyhedron vertices, six polyhedron edges, and four equivalent equilateral triangular faces is called a _____[11]. A _____[12] is a solid shape which has six congruent squares for its _____[13]. It has 8 _____[14] and 12 _____[15].

A solid shape with two identical parallel _____[16] faces and a smooth surface that joins the faces is called a **cylinder**. If that surface were flattened out, it would form a rectangle.

A_____[17] is a solid with a circle as a base and a smooth side that ends in a point. The point is called the _____[18].

A solid shape whose _____[19] is formed from all points that are a fixed distance (radius) from a fixed point (centre) is called a **sphere**.

A _____[20] is a surface of revolution generated by revolving a circle in three-dimensional space about an axis coplanar with the circle.

Exercise 4. Complete the sentences with the correct form of the verbs in brackets. Choose from the zero, first, second, or third conditional.

1) If the company _____(*to invest*) in new technology, it could increase its productivity significantly.
2) If you _____(*to study*) harder, you would have passed the exam with flying colours.
3) If we _____(*to implement*) this strategy last year, we might have seen a higher profit margin.
4) If he _____(*to save*) a portion of his income regularly, he would be financially secure now.
5) If you _____(*to attend*) the seminar, you will gain valuable insights into market trends.
6) If they _____(*to calculate*) the expenses accurately, they wouldn't have faced budgetary constraints.
7) If we _____(*to offer*) discounts on bulk purchases, we might attract more customers.
8) If the project _____(*to be*) completed on time, we could have secured the contract.
9) If she _____(*to invest*) in stocks, she might have earned substantial returns by now.
10) If they _____(*to analyse*) the data thoroughly, they will identify areas for improvement in the process.