UNIT 8
Functions

INTRODUCTION

Exercise 1. Put the words in brackets into the correct form to complete the text.

The concept of function is ________________________ (right) considered as one of the most important in all of mathematics. As the point, the line, and the plane were the basic elements of ________________ (Euclid) geometry, the dominant theory from the time of Ancient Greece until the Modern Age, the notions of function and derivative constitute the foundation of ________________ (mathematics) analysis, the theory that has become ________________ (center) in the development of mathematics since then. Particular instances of functions may be found in ancient epochs; for example, counting, implying a ________________ (to correspond) between a set of given objects and a sequence of counting numbers; the four elementary ________________ (arithmetic) operations, which are functions of two variables; and the ________________ (Babylon) tables of reciprocals, squares, square roots, cubics, and cubic roots. ________________ (history/adv), some mathematicians can be regarded as having foreseen and come close to a modern ________________ (formulate) of the concept of function. But the ________________ (to emerge) of functions in mathematics research as a clearly individualized concept and as an object of study in its own right is quite recent, dating to the end of the 17th century. The notion of function as an individualized mathematical entity can be traced to the ________________ (to begin) of infinitesimal calculus. Descartes (1596-1650) ________________ (clear) stated that an equation in two variables, ________________ (geometry) represented by a curve, indicates a ________________ (to depend) between variable quantities. The idea of derivative came about as a way of finding the tangent to any point of this curve.

Newton (1642-1727) was one of the first mathematicians to show how functions could be developed in infinite power series, thus allowing for the intervention of infinite processes. It was Leibniz (1646-1716) who first used the term "function" in 1673. He took function to designate, in very general terms, the dependence of ________________ (geometry) quantities associated with a curve. He also introduced the terms "constant," and "variable ". Euler (1707-1793) employed the word to mean any equation or formula involving variables and constants. His idea of a function is the one of most often used today in courses that precede calculus. However, the use of functions in investigating heat flow equations led to a very broad definition, due to Dirichlet (1805-1859), which described a function as a rule or ________________ (to correspond) between two sets. The ________________ (to evolve) of function has continued. In the 19th century, however, the notion of function underwent successive ________________ (to enlarge) and clarifications that deeply changed its nature and meaning.
**Exercise 2.** Fill in the table.

<table>
<thead>
<tr>
<th>VERB</th>
<th>NOUN</th>
<th>ADJECTIVE</th>
<th>ADVERB</th>
</tr>
</thead>
<tbody>
<tr>
<td>associate</td>
<td>different</td>
<td></td>
<td></td>
</tr>
<tr>
<td>correspond</td>
<td>analysis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>enlarge</td>
<td>algebra</td>
<td>X</td>
<td>dependent</td>
</tr>
</tbody>
</table>

**TERMINOLOGY**

**Exercise 3.** Read the text and match the boldfaced expressions with their descriptions below.

One of the most important aspects of any science (managerial, life, social, physical, etc.) is the establishment of correspondences, or functional relations, among various types of phenomena. Once a correspondence is known, predictions can be made. A cost analyst would like to predict costs for various levels of output in a manufacturing process; a medical researcher would like to know the correspondence between heart disease and obesity; a psychologist would like to predict the level of performance after a subject has repeated a task a given number of times; and so on. Each of these examples describes the matching of elements from one set with the elements in a second set. Any procedure or a rule that assigns to each member of one set $X$ one, and only one, element of another set $Y$ is called a function.

If $f$ is a symbol used to denote a function from a set $X$ to a set $Y$, then we write $f : X \rightarrow Y$ (arrow notation : we also say that $f$ “maps” $X$ to/onto $Y$) and call $X$ the domain of $f$ and $Y$ the codomain of $f$. If a specific element $x$ of the set $X$ is mapped to the element $y$ in $Y$, then we write $y = f(x)$ - it is read as “$f$ of $x$” or “$f$ at $x$”. Since functions are so widely used, many traditions have grown up around their use. The symbol for the input to a function (argument) $x$ is often called the independent variable because it can be assigned any of the permissible numbers from the domain. The symbol for the output (value) $y$ is called the dependent variable because its value depends on the number $x$. This terminology is intended to suggest that $x$ is free to vary, but that once $x$ has a specific value a corresponding value of $y$ is determined.

A function can be given by a formula, by a plot or graph, by an algorithm that computes it, or by description of its properties. In applied disciplines, functions are frequently specified by their tables of values or by a formula.

_ a connection between two things; the fact of two things being similar
_ the output or matched value of a function
_ an expression used to calculate a desired result
_ a number or amount whose value does not depend on the value of another element in the same mathematical expression
_ the number or value that is entered, for example, into a function machine
_ the number or value that comes out from a process
_ a fact or an event in nature or society, especially one that is not fully understood
_ a series of actions that are done in a certain way or order
Exercise 4. The notion of a function can be defined in several ways. Fill in the gaps with appropriate expressions to complete so-called rule form of its definition.

A function is a rule (process or method) that produces a ________________________________ between one set of elements, called the ________________________________, and a second set of elements, called a ________________________________ (or codomain), such that to each element in the domain there corresponds one and ________________________________ element in the range.

Exercise 5. Read the agreement on domains and ranges and find expressions having meanings similar to the following expressions:

- introduced ________________________________
- to suppose, to consider ________________________________
- unless otherwise noted ________________________________
- given ________________________________
- agreement ________________________________
- constraint ________________________________

Usually the domain of a function f is not specified; only a rule or equation defining the function is given. Such a trimming of the domain of a function is known as restriction of a function. In many cases, this will not be done. Unless stated to the contrary, we will adhere to the following convention regarding domains and ranges for functions specified by equations:

**Convention:** If a function is specified by an equation and the domain is not indicated, then we assume that the domain is the set of all real number replacements of the independent variable (inputs) that produce real values for the dependent variable (outputs). The range is the set of all outputs corresponding to input values.

**Example:** Find the domain of the function defined by: \( f(x) = \frac{3}{x^2 - 4} \)

**Solution:** The rule \( f \) tells us to divide 3 by \( x - 4 \). Division by zero is not allowed; therefore, the denominator \( x - 4 \) can never be zero, so \( x \) can never equal 2 or -2. The domain of the function \( f(x) = \frac{3}{x^2 - 4} \) is the set of all real numbers except for 2 and -2.

Exercise 6. Read the text and complete the sentences below with suitable words

**Graphs of functions**

Each function that has a real number domain and range has a graph – the graph of the ordered pairs of real numbers that constitute the function. A **Cartesian coordinate system**, also known as rectangular coordinate system, can be used to plot points and graph lines. It is formed by taking two mutually perpendicular real number lines intersecting at their origins (coordinate axes), one horizontal and one vertical, and then assigning unique ordered pairs of numbers (coordinates) to each point \( P \) in the plane.

The first coordinate (absissa) is the distance of \( P \) from the vertical axis \( y \), and the second coordinate (ordinate) is the distance of \( P \) from the horizontal axis \( x \). The coordinate axes divide the plane into 4
infinite regions called **quadrants**, numbered I to IV in an anticlockwise (counter clockwise) direction starting from the upper right. When functions are graphed, domain values are usually associated with the horizontal axis (x-axis) and range values with the vertical axis (y-axis). Thus, the graph of a function \( f \) is the graph where \( x \) is the **independent variable** or the abscissa of a point on the graph of \( f \), and \( y \) and \( f(x) \) are dependent variables or the ordinate of the point on the graph of \( f \).

1) A point is located in the first quadrant if both its abscissa and ordinate are ____________.
2) A point is located in quadrant II if its abscissa is ________________ and ordinate is ________________.
3) A point is located in quadrant III if both its abscissa and ordinate are ________________.
4) A point is located in quadrant IV if its abscissa is ________________ and ordinate is ________________.

The **vertical line test** can be used to indicate, if a given graph is a graph of a function. The test states that a graph represents a function if and only if all vertical lines intersect the graph at most once.

**Exercise 7.** Which of the following are graphs of functions? Explain why.

**Exercise 8.** Fill each gap in the text below with the correct preposition.

**Functions and their graphs**

A function specified _______1 an equation of the form \( f(x) = mx + b \) is called a **linear function**, where \( m \) and \( b \) are real numbers. The domain _______2 the linear function \( f \) consists of all real numbers. The graph of this function is a nonvertical straight line with slope \( m \) and y-intercept \( b \). A linear function is increasing if \( m > 0 \), decreasing if \( m < 0 \), and stationary if \( m = 0 \).

A **quadratic function** is one _______3 the form \( f(x) = ax^2 + bx + c \), where \( a \neq 0 \) and \( a, b, \) and \( c \) are constants and \( x \) is a variable. The constants \( a, b, \) and \( c \) are called, respectively, the quadratic coefficient, the linear coefficient and the constant term or free term. The graph _______4 the quadratic function is a U-shaped curve called a parabola. A **parabola** is set _______5 all points _______6 a plane which are an equal distance away _______7 a given point and given line. The point is called the **focus** _______8 the parabola and the line is called the **directrix**. The directrix is perpendicular _______9 the axis of symmetry _______10 a parabola and does not touch the parabola.

If the axis of symmetry _______11 a parabola is vertical, the directrix is a horizontal line.
The axis of symmetry ________ a parabola is a vertical line that divides the parabola ________ two congruent halves. The axis of symmetry always passes ________ the vertex ________ the parabola. The vertex ________ the parabola is ________ equal distance ________ the focus and the directrix.

If a > 0, the parabola opens upward, its vertex is the minimum, or the lowest point where the graph changes ________ decreasing ________ increasing and such a parabola is concave upward (convex).

If a < 0, the parabola opens downward, its vertex is the maximum, or highest point where the graph changes ________ increasing ________ decreasing and such a parabola is concave downward.

**GRAMMAR**

**Linking Words: cause and effect**

Linking devices are used to link one idea or argument to another. A common situation is when we are talking about something that happens and its result, or a *cause* and its *effect*.

**Exercise 9.** Decide whether these expressions introduce a reason clause or a result

<table>
<thead>
<tr>
<th>since</th>
<th>due to</th>
<th>results (resulting) in/from</th>
<th>owing to</th>
<th>consequently</th>
<th>as a result/as result of</th>
<th>as</th>
<th>therefore</th>
</tr>
</thead>
</table>

**Exercise 10.** Decide which parts of the following sentences describe cause (C) and which describe effect (E). Underline the connecting words.

1) The after-school program was cancelled owing to lack of interest from the students.
2) Many people are still unemployed due to the fact that the economic recovery has been slower than anticipated.
3) Since the number of students interested in that course is small, it will not be opened.
4) The publisher rejected the author’s latest work owing to the fact that the manuscript was full of errors.
5) We decided not to use that system because it was not compatible with our equipment.
6) Since high level positions are at times rather stressful, professionals can sometimes be extremely impatient.

**Exercise 11.** Use the *cause* and *effect* conjunctions to complete the gaps

1) Prices in general have risen ____________ the price of oil has risen.
2) There has been relatively low inflation ____________ consumer confidence is high.
3) ____________ the economy in this country is expanding, many companies have set up here.
4) Foreign investment has been high ____________ the tax incentives being offered.
5) Inflation was very high ____________ few companies wanted to invest.
6) ____________ the economic crisis, many companies are cutting back.
7) Dr. Smith’s course will be offered only in the winter ____________ to very low enrolment.
8) He had spent most of his time hanging around instead of revising his lessons. ____________, he had a lot of trouble answering the exam questions.
LANGUAGE PRACTICE

'make' and 'do' collocations

Exercise 12. Complete the general 'make' or 'do' rules below.

1. We always use _________ to describe indefinite activities, often with what, thing, anything, nothing, etc., and generally speaking to talk about duties, actions, obligations, and repetitive tasks.
2. We tend to use _________ when we are talking about creating or producing something, and for actions you choose to do.

Exercise 13. Which of the following words are used with 'make' and which with 'do'?

_________ a suggestion  _________ complaint  _________ a decision
_________ an exam  _________ a speech  _________ a mistake
_________ contribution  _________ research  _________ an effort
_________ an impression  _________ some work  _________ an announcement
_________ an attempt  _________ a test  _________ a comment
_________ a presentation  _________ a remark  _________ an enquiry

Exercise 14. Complete the following sentences using an appropriate form of 'make' and 'do' in correct form.

1) Everyone can learn how to _________ a chart or graph in MS Excel with this video tutorial.
2) I _________ a lot of research and I think I _________ a good job on that essay.
3) We _________ a complaint with our internet provider about their terrible service, but we still haven't heard back from them.
4) Susan _________ three suggestions and left it to John to _________ the final decision.
5) The teacher _________ a few critical comments on my essay.
6) It's difficult to _________ any predictions about the future of the economy.
7) The new company _________ a profit within its first year.
8) When I grow up, I want to have a job that allows me to _________ a lot of money.
9) Don't worry about getting everything perfect – just _________ your best.
10) We _________ business with clients in fifteen countries.
11) We're _________ a course at the local university.
12) Please excuse me – I need to _________ a phone call.
13) I think I _________ pretty well in the interview.
14) Dana _________ some good points during the meeting; I think we should consider her ideas.
15) Everyone _________ badly on the test – the highest grade was 68.
16) The company president _________ a speech about ethics in the workplace.
17) I'd like to _________ an observation about our business plan.
18) You _________ a few mistakes in your calculations – the correct total is $5430, not $4530.
19) I think I could _________ a contribution to the project.
**TEXT PRACTICE**

Exercise 15. Do you know the legend of the origin of the Cartesian coordinate system? What do you know about life and work of Rene Descartes? Can you give examples of real life applications of the coordinate system?

Exercise 16. Match an expression on the left with an appropriate expression on the right to make meaningful phrases. Then put the phrases into the gaps in the article below.

1. coordinates
2. interested in
3. locations within the graph are
4. problems concerning
5. properties of shapes
6. spent time
7. the fly's position
8. without passing

A. working on problems
B. through the same place
C. that stay the same when the shapes are stretched
D. motion and maps
E. described using \((x,y)\) pairs
F. are the foundation for graphing
G. answering unusual questions
H. in relation to where it was

**Going places with mathematicians**

The idea of representing places and spaces is very important in mathematics. Many famous mathematicians have ___________________________ that involve moving and mapping things. Rene Descartes was a French mathematician, philosopher, and anatomist who lived from 1596 to 1650. He contributed to many modern ideas, mainly those associated with science and the scientific method. One of the things that Rene Descartes is most famous for is the *Cartesian Coordinate System* that is named after him. It is interesting how Descartes came to develop Cartesian coordinates. He was lying on his bed watching a fly! Slowly, it came to him that he would be able to accurately describe the fly's position at any time by just three numbers. He had to use three numbers because the room was three-dimensional. The three numbers would describe ___________________________ to the floor and two adjacent walls.

Rene Descartes' ___________________________. Most graphs use a grid with only numbers to show positions. In the lower left corner is a point called the origin, this is the beginning of the graph and it is at the point located at \((0,0)\). The bottom line of the graph and the line at the left hand side of the graph are known as the x and y axes. The x axis runs horizontally, the y axis vertically, and ___________________________.

More than 350 years after his death, Rene Descartes' system of describing the position or location of things is still very important in many different ways. The system is used by the military and rescue services as well as air traffic controllers and map makers to accurately describe where to find people and places.
mathematician who was interested in ___________________________ was Leonard Euler who lived in the 1700’s in Switzerland and then later in Russia. He is remembered today for his work in a new branch of mathematics called topology. Topologists are mathematicians who study problems and puzzles about geometrical structures. Topology sometimes gets called 'rubber sheet geometry' - because topologists study the ___________________________ or squashed. They don't ask questions such as, How big is it? They are much more ___________________________ like: Are there any holes in it? Are all the parts connected together? Can the whole thing be separated into smaller parts? One example of what topologists might be interested in is a network - like a network of train lines that cross each other and have certain points where the lines come together and then separate. If topologists were using the map of the London Underground network they would probably be looking to see if the underground lines connect up and how many different ways you can travel to the same station ___________________________ twice.