REDUCTION AND GENERATION OF CYCLICALLY 4-CONNECTED CUBIC GRAPHS

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For an edge e of a cubic graph G denote by $G \sim e$ the homeomorph of the graph $G-e$. An edge e in a cyclically 4-connected cubic graph is removable if $G \sim e$ is cyclically 4-connected. In 1988 Andresen, Fleischner and Jackson proved that a cyclically 4-connected cubic graph of order ≥ 12 has at least $\frac{1}{5}(|E(G)|+12)$ removable edges. A reverse operation to the edge-reduction consists in subdividing two independent edges and joining the two new vertices by an edge. It transpires that the only irreducible cubic cyclically 4-connected graph is the dipole $\mathcal{D}$, or equivalently, the set of cyclically 4-connected cubic graphs can be generated from the dipole by repetitive application of the adding-edge operation. Another way to reduce/generate cyclically 4-connected cubic graphs was introduced in 1975 by Payan and Sakharovitch. It is based on vertex-removals combined with square-removals. For a vertex v of G denote by $G \sim v$ the homeomorph of $G-v$. For a 4-cycle Q denote by $G \sim Q$ a cubic graph obtained from $G-Q$ by adding two edges. One can prove that every cyclically 4-connected cubic graph reduces either to the dipole, or to K_4. Observe that $G \sim Q$ is not uniquely determined, however, one can observe that only one of the three possible reductions is essential.

The reduction statements can be used in inductive proofs of the Payan-Sakarovitch theorem, and of its strenghtenings. We demostrate the main ideas on the proof(s).