

# Quasirandom-forcing Tournaments

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We recall that a *tournament* is a directed graph having precisely one arc between each pair of its nodes. Following Chung and Graham (1991), we say that a tournament  $T$  is *quasirandom* if for every  $X \subseteq V(T)$ ,

$$\sum_{v \in X} |d^+(v, X) - d^-(v, X)| = o(n^2)$$

(in fact, they provided a number of equivalent definitions of quasirandomness of tournaments). We say that a tournament  $F$  is *quasirandom-forcing* if the following is satisfied for every tournament  $T$ . If the density of  $F$  in  $T$  is the same as the expected density of  $F$  in a random tournament, then  $T$  is quasi-random.

We recall that a tournament is *transitive* if it contains no directed cycle. Coregliano and Razborov (2017) showed that every transitive tournament on at least 4 nodes is quasirandom-forcing (see also exercise 10.44(b) in Lovász (1993)). For completeness, we recall that no tournament on 3 nodes is quasirandom-forcing (it follows, for instance, from the result of Goodman (1959)).

Regarding non-transitive tournaments on  $n$  nodes, it is known that none is quasirandom-forcing for  $n = 4$ , and Coregliano, Parente and Sato (2019) showed that there is one for  $n = 5$  (and the combination of their results and the result of Hancock, Král', Skerman and Volec (2019+) gives that it is the only one for  $n = 5$ ), and Bucić, Long, Shapira and Sudakov (2019+) observed that there is none for  $n \geq 7$ . We close the remaining case by showing that there is none for  $n = 6$ .